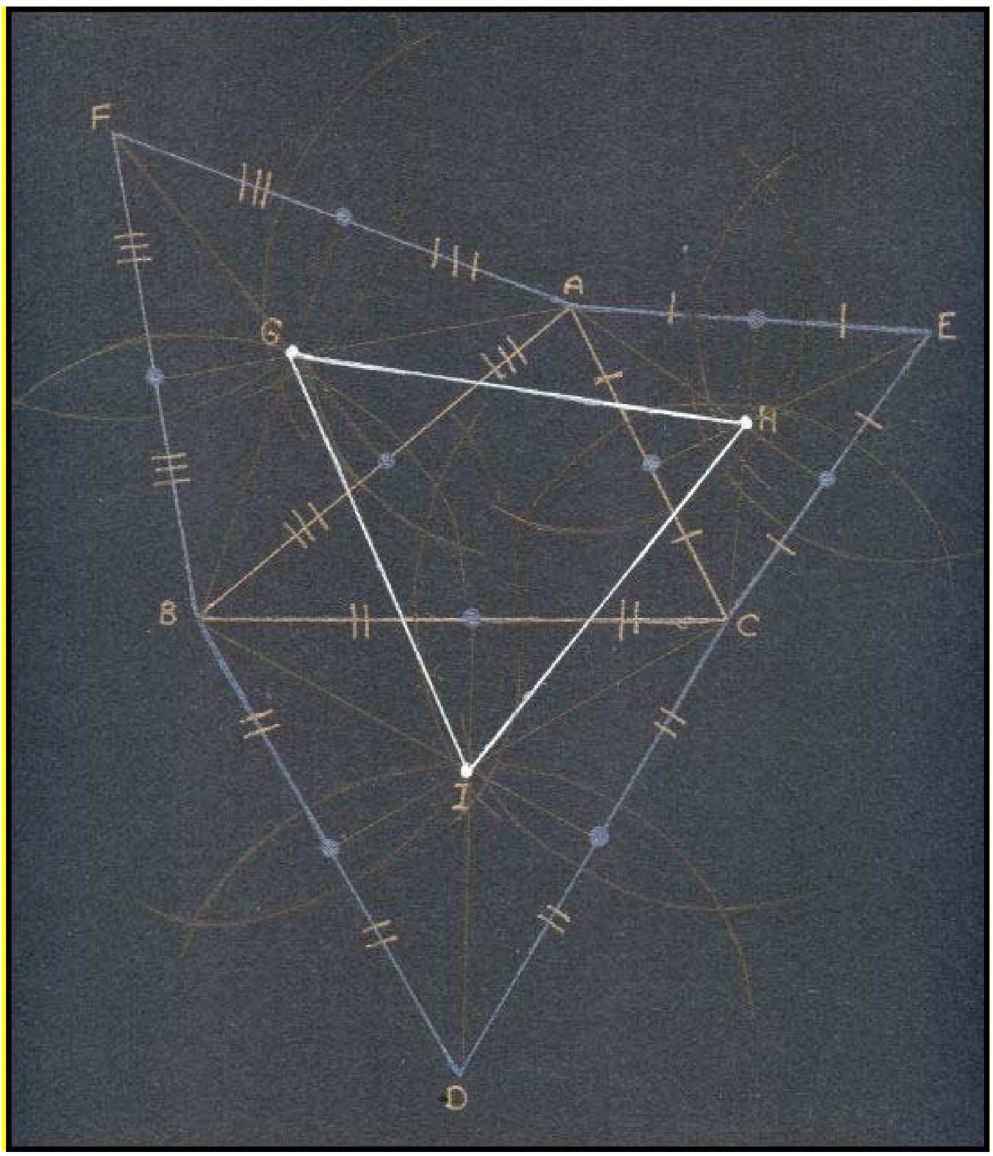
Vector

The Official Journal of the BC Association of Mathematics Teachers

Winter 2010 • Volume 51 • Issue 1



Vector **is published by the BC Association of Mathematics Teachers**

Articles and **Letters to the Editors** should be sent to:

Peter Liljedahl, *Vector* Editor
liljedahl@sfu.ca

Sean Chorney, *Vector* Editor
seanchorney@yahoo.ca

Membership Rates for 2009 - 2010

$40 + GST BCTF Member
$20 + GST Student (full time university only)
$58.50 + GST Subscription fee (non-BCTF)

Notice to Contributors

We invite contributions to *Vector* from all members of the mathematics education community in British Columbia. We will give priority to suitable materials written by BC authors on BC curriculum items. In some instances, we may publish articles written by persons outside the province if the materials are of particular interest in BC.

Articles can be submitted by email to the editors listed above. Authors should also include a short biographical statement of 40 words or less.

Articles should be in a common word processing format such as Apple Works, Microsoft Works, Microsoft Word (Mac or Windows), etc. All diagrams should be in TIFF, GIF, JPEG, BMP, or PICT formats. Photographs should be of high quality to facilitate scanning.

The editors reserve the right to edit for clarity, brevity, and grammar.

Membership Enquiries

If you have any questions about your membership status or have a change of address, please contact the BCAMT Membership Chair:

Dave Ellis (daveellis@shaw.ca)

Notice to Advertisers

Vector is published three times a year: spring, summer, and fall. Circulation is approximately 1400 members in BC, across Canada, and in other countries around the world.

Advertising printed in *Vector* may be of various sizes, and all materials must be camera ready.

Usable page size is 6.75 x 10 inches.

Advertising Rates Per Issue

Full Page $ 300
Half Page $ 160
Quarter Page $ 90

Technical Information

The layouts and editing of this issue of *Vector* were done on a Dell using the following software packages: Adobe Acrobat Professional, Adobe Photoshop, and Microsoft Word.

Vector

Winter 2010 • Volume 51 • Issue 1

ON THE COVER: Napoleon's Triangle by Rachel McNamee, a grade 10 student enrolled at Magee Secondary School in Vancouver.

The 2009–2010 BCAMT Executive

President and Newsletter Editor

Dave van Bergeyk
Salmon Arm Secondary School
Work: 250-832-2188
dvanberg@sd83.bc.ca

Past President

Michèle Roblin
Howe Sound Secondary School (Squamish)
Work: 604-892-5261
Fax: 604-892-5618
mroblin@sd48.bc.ca

Vice President

Chris Becker
Princess Margaret Secondary (Penticton)
Work: 250-770-7620
cbecker@summer.com

Secretary

Brad Epp
South Kamloops Secondary
Work: 250-374-1405
bradepp@mail.ocis.net

Treasurer

Kathleen Wagner
Hugh Boyd Secondary School (Richmond)
Work: 604-668-6615
kwagner@sd38.bc.ca

Membership Chair

Dave Ellis
Home: 604-327-7734
daveellis@shaw.ca

Elementary Representatives

Jessica Anjos
Ellison Elementary School (Kelowna)
Work: 250-870-5000
janjos@sd23.bc.ca

Jennifer Griffin
South Slope Elementary (Burnaby)
Work: 604-760-1655
jennifer.griffin@sd41.bc.ca

Carollee Norris
Numeracy Support Teacher
School District 60 (Peace River North)
Work: 250-262-6028cnorris@prn.bc.ca or
cnorris1@telus.net

Lorill Vining
Numeracy Support Teacher
School District 72 (Campbell River)
lorill.vining@sd72.bc.ca

Donna Wright
Student Achievement Helping Teacher School
School District 34 (Abbotsford)
Work: 604-859-4903
donna_wright@sd34.bc.ca

Middle School Representative

Dawn Driver
H.D. Stafford Middle School (Langley)
Work: 604-534-9285
ddriver@sd35.bc.ca

The 2009–2010 BCAMT Executive

Secondary Representatives

Michael Finnigan
Yale Secondary School (Abbotsford)
Work: 604-853-0778
michael_finnigan@sd34.bc.ca

Marc Garneau
Mathematics Helping Teacher (Surrey)
Work: 604-590-2588
piman@telus.net

Sam Muraca
District Coordinator - Numeracy
School District 35 (Langley)
Work: 604-534 7891
smuraca@sd35.bc.ca

Independent School Representative

Chris Stroud
West Point Grey Academy (Vancouver)
Work: 604-222-8750
cstroud@wpga.ca

Post-Secondary Representative

Peter Liljedahl
Simon Fraser University
Work: 778-782-5643
liljedahl@sfu.ca

NCTM Representative

Marc Garneau
Mathematics Helping Teacher
School District 36 (Surrey)
Work: 604-590-2588
piman@telus.net

Vector Editors

Peter Liljedahl
Simon Fraser University
Work: 778-782-5643
liljedahl@sfu.ca

Sean Chorney
Magee Secondary School (Vancouver)
Work: 604-713-8200
schorney@vsb.bc.ca

LETTER FROM THE EDITORS

Under the editorialship of David Tambellini and John Kamimura *Vector* maintained its distinction as an award winning journal produced **by** mathematics teachers **for** mathematics teachers. With the passing of the editorialship to Peter Liljedahl and Sean Chorney in the Fall of 2009 *Vector,* heads into a new era – an era which will see the high standards established by David and John maintained, along with a few changes.

Over the last few years the BCAMT has become a place for teachers of mathematics from K to 12 to come together. They look upon BCAMT not only as an organizing body but also a resource. The BCAMT conference, website, listserve, newsletter, and journal have become the venues for the dissemination of resources. As such, Vector has been, and will continue to be a tremendous tool for mathematics teachers of this province. We hope to sharpen this tool. Mobilizing a vast network of teachers, teacher educators, and researchers we hope to make Vector even more responsive to the needs of K to 12 teachers in this province. As such, you can expect to see an emergence of special interest columns within the journal – columns managed by teams of assistant editors. In this issue some of these columns are already starting to take shape. However, we will not draw special attention to them yet. Once they begin to stabilize and regular editorial teams are established we will announce them formally. Until then, please enjoy the current edition in its unstable form.

CALL FOR SUBMISSIONS

We are looking for quality submissions of the following:

- research reports
- literature reviews
- stories of teaching
- teacher resources
- relevant website links
- interesting problems
- students' solutions to problems
- book reviews
- letter to the editor

Articles can be submitted by email to the editors listed on page 2. Authors should also include a short biographical statement of 40 words or less. Articles should be in a common word processing format such as Apple Works, Microsoft Works, Microsoft Word (Mac or Windows), etc. All diagrams should be in TIFF, GIF, JPEG, BMP, or PICT formats. Photographs should be of high quality to facilitate scanning. The editors reserve the right to edit for clarity, brevity, and grammar.

A Primary Teachers' Study Group: Investigating the Use of Children's Literature in the Teaching and Learning of Mathematics

by Janice Novakowski

Janice Novakowski is a District Curriculum Coordinator in the Richmond School District and has submitted this article on behalf of the Richmond Primary Teachers Study Group. The study group meets regularly to discuss mathematical thinking and problem-solving. They were awarded a grant from the BCAMT which enabled them to purchase picture books and develop lessons, such as the one included in this article.

The Primary Teachers Study Group is a group of Richmond teachers that meets approximately every six weeks to share ideas from their classrooms and engage in personally chosen professional development. This year our group chose to focus on non-fiction literacy and communication of thinking during mathematical problem solving. We were fortunate to receive a grant from the BCAMT and purchased several children's books from which we developed problem-based mathematics lessons. During these lessons, we focused on how students communicated their thinking both orally and with pictures, numbers and/or words. The use of non-fiction text features, such as text boxes, labels and captions, were utilized as students communicated in their math journals. An example of one of our lessons follows.

The tasks and problems we engaged with were intended to be open-ended and provide opportunities for all students to access and be engaged with mathematics. When we met and shared the problems we had been working on with our students, teachers often commented that *all their students were successful* and that these activities were just what we want in our classrooms?

If you are interested in ordering a booklet of lessons created by the study group this year, please contact Janice Novakowski (jnovakowski@sd38.bc.ca) for more information.

Primary Teachers Study Group 2008-2009

Joan Court	Marla McPherson
Kim Dunnigan	Anna Nachbar
Jacqueline Flewelling	Penny Nakamoto
Katherine Myers	Louesa Newman
Michelle Hikida	Janice Novakowski
Shauna Hudson	Romina Park
Nichole Kusch	Lisa Sameshima
Sarah Loat	Lisa Schwartz
Bonnie Froh	Jamie-Lynn Valiquette
Jordan McCurrach	Sasha Wise

Communicating mathematical thinking

As many schools begin to implement the new Mathematics K-7 IRP, teachers are thinking about ways to focus on mathematical processes. The two processes we chose to focus on as a study group were communication and problem-solving.

Students first describe their thinking verbally as a way to sort out their thoughts and were then encouraged to record their thinking with words, pictures and numbers.

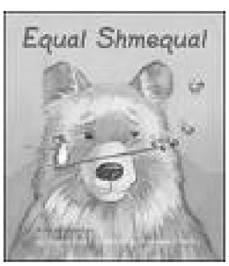
Some oral communication structures that were successfully during mathematical problem-solving lessons include: partner talks (think-pair-share), a sharing circle at the end of the lesson, exploration using speech and thinking bubbles to capture student thinking as they work on math problems. Older students could record their own thoughts while teachers scribe what students verbalize.

We have also explored how students record their thinking using non-fiction text features such as textboxes, titles, labels, captions, etc. and one can see this reflected in the student examples in the following lesson.

CREATING BALANCE

Lesson Background:

The students had been working on measuring the masses of objects using pan balances (estimating, weighing, comparing and ordering). This lesson provides an introduction to the algebraic thinking and the concepts of equality and inequality as they relate to balancing.



Equal Shmequal
by Virginia Kroll

Summary: A group of forest animals try to figure out how to create equally balanced teams for a game of tug of war.

> Animal Values
> The teacher or students can assign values to the different animals (ie the bear could be 100kg or "worth" 20). If you want the animal values to correspond to the illustrations in the story so that Bear and Mouse "equal" the rest of the animals, the following values can be used: Bear=22, Deer=9, Wolf=5, Bobcat=4, Rabbit=3, Turtle=2 and Mouse=1.

Problem:

How many different ways can you balance the animals on the seesaw?

Lesson Overview:

- In preparation for the lesson, students were given opportunities to play with materials and balances to develop the concepts of "balance" and "equality". This involved weighing and comparing different objects and sharing their informal findings with each other.
- As the story Equal Shmequal was read, we discussed what was fair and equal with students making personal connections.
- Students could be given pictures of the animals and possible assigned values when the problem is posed. Using resources such as pan balances, cubes or number balances, students were asked to represent their solutions in their math notebooks. As students worked in pairs, they talked about how to solve the problem and ways to show their ideas. Questions such as the following were asked: *How do you know these are equal?*
- Students shared the different animal combinations that made equal teams.

IMPORTANT NOTE: If students are going to be provided with the assigned values that correspond with the story, there is only one way to balance the teams using all of the animals. If you choose this option, stop reading the story before it reveals that Bear and Mouse will be on one team and all the other animals will balance them.

Other Possibilities:

- What new combinations could we make if a moose joined the group? What would its approximate value be?
- What new combinations could we make if more than one mouse or turtle (etc) could be used?
- What is equal?

Teacher's Thoughts:

Students enjoyed the story and nearly all were able to come up with more than one combination of animals that balanced. Working together encouraged them to ask questions and appreciate a variety of ways to solve this problem. Students for whom the problem was challenging were able to use fewer animals with lower values. I found that for some, explaining their math thinking verbally is still a challenging skill that will need to continue to be modeled and encouraged.

- Penny Nakamoto
(gr. 1/2 - Kidd Elementary)

Overall, I felt the lesson was very successful. It took many of the students a bit of time to come up with a plan and a strategy. I found the visual of the unifix cubes built into rods was very helpful for the students as they were able to make a logical conclusion that the bear was the biggest and would be a good starting point as they worked on the problem. Our previous practice with number balances really paid off for many of the students. They were able to methodically add weights to the scale until it was balanced.

- Louesa Newman
(gr. 1/2 - Tait Elementary)

Equal Shmequal is a fun book that helps the students visualize equality. The students relate to the story because they are constantly seeking "equal" teams at recess and lunch. The lesson was open-ended (given animal values but asked to come up with all sorts of different balanced teams) so that all the students could be successful and challenged. The students were very engaged and enjoyed solving the problem.

Anna Nachbar
(gr 2/3 - McNeely Elementary)

Student Responses

Students in grades 2 and 3 at Blair Elementary were provided with balances and asked to explore the balancing problem. The teachers listened to the mathematical language that emerged such as *balance, fair, the same as and equal* as students placed different materials on each side of the pan balances.

- A grade 2 student at Tait uses a number balance to make his teams equal. He puts the bear (22) on one side and keeps adding weights to the other to make it equal.
- A grade 1 student at Tait builds a tower of Unifix cubes for each animal (ie Wolf= 5) and then moves them around to make equal teams.
- A grade 2 student from Tait used Unifix cubes snapped together to balance the two teams. She said, "I kept counting until I added to 23."
- A grade 3 student from McNeely created a variety of "equal" teams using the assigned values for the animals in the book. The student has used coding to create the equations (W=wolf) which is a "big" idea in algebra. This student wrote the numeric value beside each letter (B22) while some students in the class were able to let the letter stand alone for the value.
- One student in the class asked if they could use more than one of each animal, to which the teacher responded yes, which opened up the problem even more for the students.

Some Ways Students Explained their Thinking:

I know it's equal because I added the numbers in my head.

I memorized the numbers (assigned values). Then I looked for different ways to make them equal.

I put the cubes for each of the animals in the bucket.

I just used cubes. When they added up to the same, I knew they were equal.

I wrote the numbers down. Then I could just add them up.

- Grades 1 and 2, Kidd Elementary

A Math 11 Diversion

by Tony Crossley and Paul Collis

Tony Crossley and Paul Collis teach at Brentwood College School in Mill Bay.

I recently missed one of my Math 11 classes. Kindly stepping in to supervise was my esteemed literate but innumerate colleague Mr. Paul Collis, head of our English department. As part of the work to be covered, the class was given problem # 74, page 490 in the Mathpower 11 textbook.

> *Two tangents intersect a circle at opposite ends of a diameter. How are the slopes of the tangent related? Explain.*

I thought BCAMT members would be interested in Paul's model "answer" that he shared with the students.

It is clear from the symbolic division suffered by the two tangents – cruelly separated by a void, their isolation made manifest by the gaping disconnection between them – have lost the familial bond that once connected them by blood. We know from the author's clues that the tangents are "related" (490), but only tangentially so. Therefore the connection is not father to son, or husband to wife; it stands to reason that we are looking at a more slender branch of the genealogical tree: uncle to nephew perhaps, or cousin to cousin. As always, the answer lies not in the narrow world of limiting numbers, but in the flexible realm of language. Let me explain. These two estranged family members live at opposite ends "*of a diameter*" (490) [italics mine]. They are not lost in an urban jungle or a remote plain; the setting is clear: a hillside, or slope, in a "circle" (490). One of the primary denotations of *circle* is globe, but the globular area that we have here is more than just a random orb; it is synecdochal for the world itself. Therefore, where on earth else but Shakespeare's iconic Globe Theatre? All the world is a circle and, as we know, a stage.

So where in the history of Shakespeare's Globe Theatre stage do we find a slope? Let us consider real estate. Estates are at their prime with an ocean panorama, and the heightened rhetoric of this question suggests such an environment. So we need wealth, and the seaside, and a picturesque scene that encircles – or englobes – two warring distant relations:

> *Two households, both alike in dignity...*
>
> *In fair Verona where we lay our scene. (Romeo and Juliet Prologue 1-2).*
>
> *Opposite ends? Witness the "ancient grudge" (Prologue 3) of the two families. Seaside setting? "Fair Verona" [italics mine]. Granted, the lines are in iambic pentameter, not diameter, but I consider that to be a typo.*
>
> *In short, the slopes of the tangent are related by marriage after Romeo and Juliet secretly elope to bring the warring Montague and Capulet families together.*

Conclusion? Ask an English teacher to cover Math 11 and the result inevitably ends in tragedy.

A FURTHER NOTE ON THE GOLDEN RATIO τ

by Clive Reis

Clive Reis is a semi-retired mathematics professor living in Victoria.

In his article (Now that's what I call alternative base representation, *Vector*, Summer 2008), Egan Chernoff sets $\tau = \frac{1+\sqrt{5}}{2}$ and proceeds to express positive and negative integral powers of τ in the form $a\tau + b$. In this note we show that it is in fact quite easy to express $p(\tau) = \sum a_i \tau^i$ in the form $a\tau + b$. We also obtain a formula for τ^{-n} and derive a well-known formula for $F(n)$, the *n*th Fibonacci number. Finally, we obtain the golden ratio $\tau = \frac{1+\sqrt{5}}{2}$ as two distinct limits.

EXPRESSING $\boldsymbol{p(\tau) = \sum a_i \tau^i}$ IN THE FORM $\boldsymbol{a\tau + b}$.

LEMMA: τ is a root of $x^2 - x - 1 = 0$.

Proof: Let the quadratic equation of which τ is a root be $ax^2 + bx + c = 0$. Then using the formula for the solution of this quadratic, namely $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, we see that $a = 1, b = -1,$ and $c = -1$. Hence τ is a root of $x^2 - x - 1 = 0$. Let σ be the other root of $x^2 - x - 1 = 0$ ($\sigma = \frac{1-\sqrt{5}}{2}$). Then $\tau + \sigma = 1$ and $\tau\sigma = -1$ since $x^2 - x - 1 = (x-\tau)(x-\sigma)$. It follows that $\sigma = -\tau^{-1}$ and so $\tau^{-1} = \tau - 1$.

THEOREM: Let $p(x)$ be a polynomial with real coefficients. Divide $p(x)$ by $x^2 - x - 1$ to get the quotient $q(x)$ and remainder $r(x) = ax + b$. Then $p(\tau) = a\tau + b$.

Proof: $p(x) = (x^2 - x - 1)q(x) + r(x)$. Hence
$p(\tau) = (\tau^2 - \tau - 1)q(\tau) + r(\tau) = a\tau + b$ since $\tau^2 - \tau - 1 = 0$.

Remark: Since σ is a root of $x^2 - x - 1 = 0$, it follows that $p(\sigma) = a\sigma + b$.

Example: We calculate $\tau^4 - 2\tau^3 + 3\tau^2 + \tau - 1$. Using the division algorithm for polynomials, we get $x^4 - 2x^3 + 3x^2 + x - 1 = (x^2 - x - 1)(x^2 - x + 3) + 3x + 2$ and so $\tau^4 - 2\tau^3 + 3\tau^2 + \tau - 1 = 3\tau + 2$.

A FORMULA FOR τ^{-n}

Let $F(0)=0,\ F(1)=1,\ F(n)=F(n-1)+F(n-2)$ be the Fibonacci sequence starting with the 0^{th} term. We have the following result.

THEOREM:

$$\tau^n = F(n)\tau + F(n-1) \text{ for all } n \ge 1 \text{ and } \tau^{-n} = (-1)^{n+1}F(n)\tau + (-1)^n F(n+1) \text{ for all } n \ge 1.$$

Proof: $\tau^1 = F(1)\tau + F(0)$ and $\tau^2 = F(2)\tau + F(1)$.

Assume inductively that

$$\tau^{n-1} = F(n-1)\tau + F(n-2) \text{ for some } n \ge 3.$$

Hence

$$x^{n-1} = (x^2 - x - 1)q(x) + F(n-1)x + F(n-2).$$

Thus

$$x^n = (x^2 - x - 1)q(x)x + F(n-1)x^2 + F(n-2)x.$$

But

$$F(n-1)x^2 + F(n-2)x = F(n-1)(x^2 - x - 1) + [F(n-1) + F(n-2)]x + F(n-1).$$

$$= F(n-1)(x^2 - x - 1) + F(n)x + F(n-1).$$

Hence

$$x^n = (x^2 - x - 1)[q(x)x + F(n-1)] + F(n)x + F(n-1).$$

It now follows that

$$\tau^n = F(n)\tau + F(n-1).$$

By induction

$$\tau^n = F(n)\tau + F(n-1) \text{ for all } n \ge 1.$$

Now recall that $\sigma^2 - \sigma - 1 = 0,\ \sigma = -\tau^{-1}$ and $\sigma = 1 - \tau$. Since the result just obtained is based on the fact that τ is a root of $x^2 - x - 1 = 0$, it follows that

$$\sigma^n = F(n)\sigma + F(n-1).$$

Hence

$$(-\tau^{-1})^n = F(n)(1-\tau) + F(n-1) = -F(n)\tau + F(n) + F(n-1).$$

Therefore

$$(-1)^n \tau^{-n} = -F(n)\tau + F(n+1)$$

or

$$\tau^{-n} = (-1)^{n+1}F(n)\tau + (-1)^n F(n+1).$$

A FORMULA FOR $F(N)$, THE NTH FIBONACCI NUMBER.

THEOREM: $F(n) = \dfrac{\tau^n - \sigma^n}{\sqrt{5}}$ for all $n \geq 0$.

Proof: $\tau^n = F(n)\tau + F(n-1)$ and $\sigma^n = F(n)\sigma + F(n-1)$.

Subtracting, we get

$$\tau^n - \sigma^n = F(n)(\tau - \sigma) \text{ and so } F(n) = \frac{\tau^n - \sigma^n}{\tau - \sigma}.$$

But

$$\tau - \sigma = \frac{1+\sqrt{5}}{2} - \frac{1-\sqrt{5}}{2} = \sqrt{5}.$$

Hence

$$F(n) = \frac{\tau^n - \sigma^n}{\sqrt{5}} \text{ for all } n \geq 0.$$

THE GOLDEN RATIO EXPRESSED AS 2 LIMITS.

Last, we obtain the golden ratio $\tau = \dfrac{1+\sqrt{5}}{2}$ as two different limits. As proved previously, we have $\tau^{-n} = (-1)^{n+1}F(n)\tau + (-1)^n F(n+1)$ where $F(n)$ is the nth Fibonacci number. From this we deduce the following theorem.

THEOREM: $\tau^{-k} - \tau^{-(k+1)} = (-1)^k[F(k+3) - F(k+2)\tau]$.

Proof:

$$\begin{aligned}\tau^{-k} - \tau^{-(k+1)} &= (-1)^{k+1}F(k) + (-1)^k F(k+1) - [(-1)^k F(k+1)\tau + (-1)^{k+1}F(k+2)] \\ &= (-1)^k[F(k+1) + F(k+2) - (F(k) + F(k+2))\tau] \\ &= (-1)^k[F(k+3) - F(k+2)\tau]\end{aligned}$$

The fact that τ is a solution to $x^2 - x - 1 = 0$ yields $\tau^2 = \tau + 1$. It follows, by multiplying both sides by τ^{-1}, that $\tau = \tau^{-1} + 1$, or $\tau^{-1} = \dfrac{1}{1+\tau^{-1}}$. Since τ^{-1} is less than 1, we have

$$\tau^{-1} = \sum_0^\infty (-1)^k \tau^{-k} = (1 - \tau^{-1}) + (\tau^{-2} - \tau^{-3}) + (\tau^{-4} - \tau^{-5}) + ...$$

and so

$$\tau^{-1} > (1 - \tau^{-1}) + (\tau^{-2} - \tau^{-3}) + ... + (\tau^{-2k} - \tau^{-(2k+1)}) \text{ since } \tau^{-2t} - \tau^{-(2t+1)} > 0 \text{ for all } t.$$

By the previous theorem

$$F(3)+F(5)+...+F(2k+1)-[F(2)+F(4)+....+F(2k)]\tau<\tau^{-1}.$$

Hence

$$(1)\qquad \frac{F(3)+F(5)+....+F(2k+1)}{F(2)+F(4)+....+F(2k)}<\tau+\frac{\tau^{-1}}{F(2)+F(4)+...+F(2k)}$$

Similarly, we can think of the infinite series above as

$$\tau^{-1}=1-(\tau^{-1}-\tau^{-2})-(\tau^{-3}-\tau^{-4})-...$$

and so

$$\tau^{-1}<(1-\tau^{-1})+(\tau^{-2}-\tau^{-3})+...(\tau^{-2k}-\tau^{-(2k+1)})+\tau^{-(2k+2)}.$$

As before

$$\tau^{-1}<F(3)+F(5)+...F(2k+1)-[F(2)+F(4)...F(2k)]\tau+\tau^{-(2k+2)}.$$

Hence

$$\frac{F(3)+F(5)+...F(2k+1)}{F(2)+F(4)+...+F(2k)}>\tau+\frac{\tau^{-1}}{F(2)+F(4)+...F(2k)}-\frac{\tau^{-(2k+2)}}{F(2)+F(4)+...+F(2k)}.$$

Combining this inequality with inequality (1) and using the fact that

$$\frac{\tau^{-1}}{F(2)+F(4)+...+f(2k)}\to 0 \text{ and } \frac{\tau^{-(2k+2)}}{F(2)+F(4)+...+F(2k)}\to 0 \text{ as } k\to\infty$$

we arrive at

$$\frac{F(3)+F(5)+...F(2k+1)}{F(2)+F(4)+...+F(2k)}\to\tau \text{ as } k\to\infty.$$

Note: Adding $F(1)$ to the numerator will not affect the limit since

$$\frac{F(1)}{F(2)+F(4)+...+F(2k)}\to 0 \text{ as } k\to\infty.$$

Therefore we may write

$$\frac{F(1)+F(3)+...+F(2k+1)}{F(0)+F(2)+...+F(2k).}\to\tau \text{ as } k\to\infty.$$

In other words, the sum of odd F's divided by the sum of the same number of even F's tends to τ.

THEOREM: (1) $F(1)+F(3)+\cdots+F(2k-1)=F(2k)$ and

(2) $F(0)+F(2)+\cdots+F(2k)=F(2k-1)-1.$

Proof: (1) When $k = 1$, the equality holds. Assume inductively that the equality holds for some $k>0$: i.e., assume

$$F(1)+F(3)+\cdots+F(2k-1)=F(2k).$$

Then

$$F(1)+F(3)+...+F(2k-1)+F(2k+1)=F(2k)+F(2k+1)=F(2k+2).$$

By induction, the result follows. A similar induction establishes (2).

COROLLARY: As $k\to\infty$, $\frac{F(2k+2)}{F(2k+1)}\to\tau$.

Proof: Since

$$\frac{F(1)+F(3)+...+F(2k+1)}{F(0)+F(2)+...+F(2k).}\to\tau \text{ as } k\to\infty$$

we get

$$\frac{F(2k+2)}{F(2k+1)-1}\to\tau \text{ as } k\to\infty.$$

and so

$$\frac{F(2k+1)-1}{F(2k+2)}\to\tau^{-1} \text{ as } k\to\infty.$$

But

$$\frac{1}{F(2k+2)}\to 0 \text{ as } k\to\infty$$

and so

$$\frac{F(2k+1)}{F(2k+2)}\to\tau^{-1} \text{ as } k\to\infty.$$

The result now follows by inverting.

A Second Look at Multipliers Algorithm for Solving Polynomial Equations and Factoring

By Ali Astaneh

Ali Astaneh is a mathematics teacher at Prince of Wales Secondary in Vancouver.

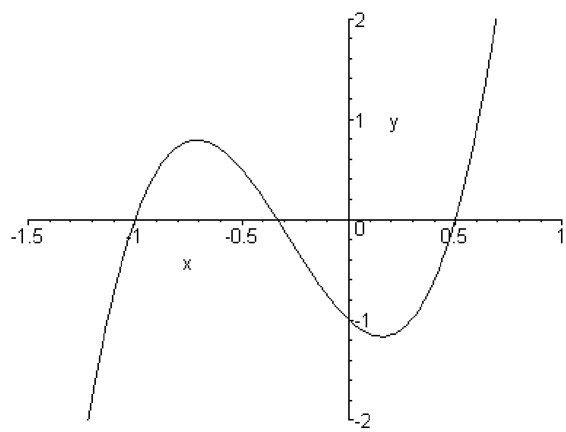
Early in 2003, while I was trying to address questions posted on the BCAMT list-serve from math teachers, I encountered a method (which I call *multipliers algorithm*) to find rational zeros of a polynomial function without using the rational zero theorem. It turned out to be very interesting. So interesting that a senior UBC math professor on the list made the following comment about it:

> *...The factoring algorithm you have described in various earlier postings is very nice, and is certainly a significant improvement on the procedures ordinarily taught...*

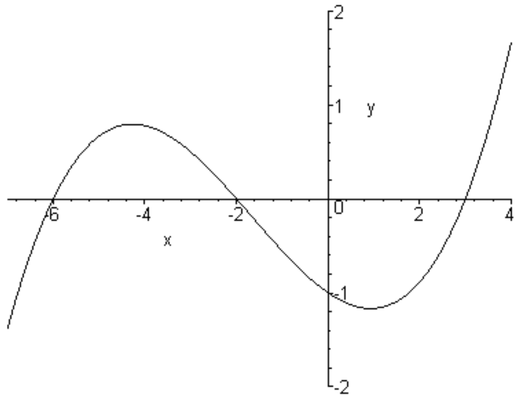
In an attempt to compensate the lack of publicity of the method and to include the many new math teachers that have come along to teach in B.C. secondary schools since 2003, I recently gave a presentation about the method in February 2009 Vancouver Math Conference at Prince of Wales Secondary for the second time; the first time being in the Fall 2003 North West Conference at Whistler. The other reason for my February presentation was to give an amazingly simpler analytical geometric account of the solution for finding rational zeros of a polynomial, before presenting the algebraic algorithm.

Since two of the math teachers attending both Whistler and Vancouver presentations told me the second time in February was much easier to understand and pleasant to follow, I thought I should revise the presentation handout and turn into a short article for the attention of interested math teachers.

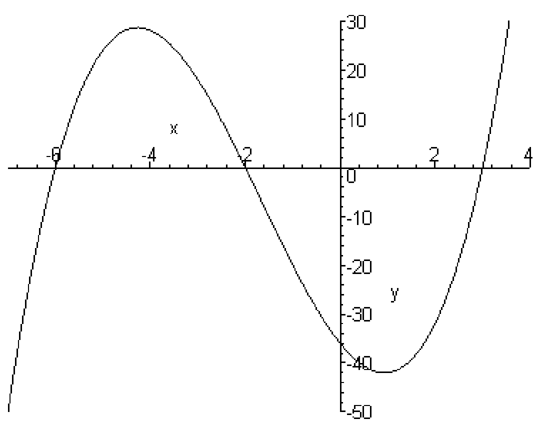
To start with, the idea behind the method is amazingly simple. Assume for the sake of argument you have a polynomial function with integer coefficients $p(x)$ which has say only three rational zeros $-1, -\frac{1}{3}$, and $\frac{1}{2}$. [Note that the software used for this article labels rational numbers on the *x*-axis as decimals; that is, for example, 0.5 means $\frac{1}{2}$]



If you multiply these three rational zeros by their common denominator 6, you obtain three integers $-6, -2$, and 3 *which happen to be* the zeros of horizontal expansion of the polynomial $p(x)$ by a scale of 6; that is the polynomial $p(x/6)$. Except that the coefficients of this new polynomial may be non-integer rational numbers.



However, you can always vertically expand the new polynomial $p(x/6)$ by a positive integer *M*, so that the coefficients of the new polynomial $P(x) = M\,p(x/6)$ all become integers; while at the same time the integers $-6, -2,$ and 3 remain the three zeros of $P(x)$. This is because a vertical expansion never changes zeros of a polynomial function. We will shortly see why that appropriate integer *M* best be chosen 6^2 ; that is [the leading coefficient of the original *p*(*x*)] raised by [degree of *p*(*x*) – 1].



Here, the advantage about working with $P(x)$ will be that you only need to use the integral zero theorem to find its integer zeros. Then you can recover the required zeros of the original polynomial $p(x)$ by dividing those integer zeros of $P(x)$ all by 6.

The following example is the precise algebraic translation of the analytical geometric approach described above.

Example 1: Let's say you would like to find rational zeros of the polynomial function

$$p(x) = 6x^3 + 5x^2 - 2x - 1.$$

If you horizontally expand this polynomial by its leading coefficient 6, as mentioned above the zeros of the new expanded polynomial $p(x/6) = \frac{1}{36}x^3 + \frac{5}{36}x^2 - \frac{1}{3}x - 1$ will be exactly 6 times zeros of $p(x)$. Since vertical expansion of a polynomial will never change its zeros, it follows that zeros of the polynomial

$$P(x) = 6^2\,p(x/6) = x^3 + 5x^2 - 12x - 36$$

will also be 6 times zeros of the original polynomial $p(x)$. Note that here the vertical scaling factor 6^2 is to be interpreted as the leading coefficient of $p(x)$ raised by the integer (degree of $p(x)$ -1) . One can now use the integral zero theorem, and factor the auxiliary polynomial $P(x)$ as follows

$$P(x) = (x-3)(x+6)(x+2).$$

Since the three zeros of $P(x)$ are $x = -6, -2, 3$ we obtain the three rational roots of the original polynomial $p(x)$ as

$$x = -\frac{6}{6}, -\frac{2}{6}, \frac{3}{6}$$

which means $x = -1, -\frac{1}{3}, \frac{1}{2}$.

Note, that rational zero theorem was put out of business here; and that is what I meant by " redundancy of rational zero theorem in dealing with polynomial equations and factoring“ in the title of my February presentation.

Since the problem of finding rational zeros of a polynomial is in the PMath11 curriculum in B.C., and at that level students haven’t learned about horizontal and vertical expansion transformations of functions, I later converted the above analytical geometric approach into an algebraic algorithm, without any mention of horizontal or vertical expansion of polynomials. For the obvious reasons we shall see shortly I called the method “ The Multipliers Algorithm”. I will now bring a second example to explain how we go through the three steps of the algorithm.

Example 2: Let's say you would like to factor the polynomial

$$p(x) = 6x^3 + 13x^2 + x - 2 .$$

Since there are four coefficients for this polynomial, consider the four ascending powers $6^{-1}, 6^0, 6^1, 6^2$ of the leading coefficient 6 of the polynomial, and follow these steps:

Step1 Multiply each one of the four ascending powers $6^{-1}, 6^0, 6^1, 6^2$ in order by the respective coefficients (from left) of the original polynomial $6x^3 + 13x^2 + x - 2$ to obtain a new auxiliary polynomial, which means

$$P(x) = x^3 + 13x^2 + 6x - 72 .$$

Remark: This Step1 will horizontally expand the polynomial $p(x)$ by 6 and then vertically expand the result by 6^2 at the same time to obtain the auxiliary polynomial

$$P(x) = 6^2 p(x/6)$$

whose roots are 6 times those of $p(x)$ with integer coefficients.

Step2 Use the integral zero theorem to factor the auxiliary polynomial $P(x)$ as follows (to save time when doing so, only examine those factors of the constant term -72 whose absolute values don't exceed $|6(-2)| = 12$). Then

$$P(x) = x^3 + 13x^2 + 6x - 72 = (x-2)(x+3)(x+12) .$$

Step3 On the right hand side of the above equation in Step2, replace each bracket $(x - r)$ by the bracket $(\frac{6}{g}x - \frac{r}{g})$ where g is the greatest common factor between the original leading coefficient 6 and the root r. Then the required factorization

$$(3x-1)(2x+1)(x+2)$$

of the original polynomial

$$p(x) = 6x^3 + 13x^2 + x - 2$$

is obtained.

The Multipliers Algorithm in General

To factor the polynomial

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + \ \dots \ + a_1 x + a_0$$

with integer coefficients and rational roots, consider the $(n+1)$ ascending powers of the leading coefficient $a_n^{-1}, a_n^{0}, a_n^{1}, \dots\ a_n^{n-2},\ a_n^{n-1}$ and follow these steps:

Step 1 Multiply each of the $(n+1)$ coefficient $a_n, a_{n-1}, a_{n-2}, \dots\ a_1,\ a_0$ of the polynomial $p(x)$ by the respective ascending powers of the leading coefficient a_n, that is by $a_n^{-1}, a_n^{0}, a_n^{1}, \dots\ a_n^{n-2},\ a_n^{n-1}$ to obtain a new auxiliary polynomial

$$P(x) = x^n + c_{n-1}x^{n-1} + \ \dots \ + c_1 x + c_0$$

Step 2 Use the **integral zero theorem** to factor the auxiliary polynomial $P(x)$, say as follows (when doing so only test only those factors of the constant term

$$c_0 = a_n^{\,n-1} a_0$$

whose absolute values don't exceed $|a_n a_0|$). (*)

$$x^n + c_{n-1}x^{n-1} + \ \ldots \ + c_1 x + c_0 = (x - r_1)(x - r_2) \ \ldots \ (x - r_n)$$

Here $r_1, r_2, \ldots, r_n$ denote all integer zeros of $P(x)$.

Step 3 For $k = 1, 2, \ \ldots \ n$ let g_k be the greatest common factor between the original leading coefficient a_n, and the root r_k in the above equation (*). Then replace each bracket $(x - r_k)$ in (*) by the

bracket $(\frac{a_n}{g_k} x \ - \ \frac{r_k}{g_k})$. Then the required factorization of the original polynomial is obtained as follows

$$a_n x^n + a_{n-1} x^{n-1} + \ \ldots \ + a_1 x + a_0 =$$

$$(\frac{a_n}{g_1} x \ - \ \frac{r_1}{g_1})(\frac{a_n}{g_2} x \ - \ \frac{r_2}{g_2}) \ \ldots \ (\frac{a_n}{g_n} x \ - \ \frac{r_n}{g_n})$$

Remark: Note that in the above algorithm I have assumed that common factoring already has been done. That is, I have assumed that there is no common integer factor (other than 1) between all the coefficients of a given original polynomial $a_n x^n + a_{n-1} x^{n-1} + \ \ldots \ + a_1 x + a_0$. In case there is one, and say there is a greatest common factor $C > 1$ between coefficients, then we will factor the polynomial as $C\,[a'_n x^n + a'_{n-1} x^{n-1} + \ \ldots \ + a'_1 x + a'_0]$ and then apply the algorithm to the polynomial

$$p(x) = a'_n x^n + a'_{n-1} x^{n-1} + \ \ldots \ + a'_1 x + a'_0$$

Proof: Given the original polynomial

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \ \ldots \ + a_1 x + a_0,$$

as mentioned in the above remark, we can assume (without loss of generality) that the coefficients of $p(x)$ have no positive integer factor in common other than 1. Also, without loss of generality, we will assume that the leading coefficient a_n of $p(x)$ is a positive integer. Then the polynomial $P(x)$ defined in Step 1 of the algorithm is the same as

$$P(x) = x^n + c_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + c_1 x + c_0$$
$$= a_n^{\,(n-1)} p(x / a_n)$$

where $c_k = a_n^{\,n-(k+1)} a_k$ for $k = 1, 2, \ \ldots \ n$. On the other hand since zeros of $P(x)$ are just a_n times zeros of the polynomial $p(x)$, the latter polynomial will have a rational zero if and only if $P(x)$ has an integral zero. More precisely, an integer r will be a zero of $P(x)$ if and only if r / a_n a zero of the polynomial is $p(x)$.

Suppose now that, after using the integral zero theorem, the polynomial $P(x)$ can be factored as

$$P(x) = (x - r_1)(x - r_2) \ \ldots \ (x - r_n)$$

where $r_1, r_2, \ldots, r_n$ are all integers.

Since $P(x) = a_n^{\,(n-1)} p(x / a_n)$ implies $p(x) = (1 / a_n^{\,(n-1)}) P(a_n x)$; assuming that for $k = 1, 2, .., n$ the greatest common factor between the leading coefficient a_n and the zero r_k is denoted by g_k, we have

$$p(x) = (1 / a_n^{\,(n-1)}) P(a_n x)$$

$$= (1/a_n^{(n-1)})(a_n x - r_1)(a_n x - r_2)\ \ldots\ (a_n x - r_n)$$
$$= \frac{g_1 g_2\ \cdots\ g_n}{a_n^{(n-1)}}\left(\frac{a_n}{g_1}x - \frac{r_1}{g_1}\right)\left(\frac{a_n}{g_2}x - \frac{r_2}{g_2}\right)\ \ldots\ \left(\frac{a_n}{g_n}x - \frac{r_n}{g_n}\right)$$

Since each of the brackets $(\frac{a_n}{g_k}x - \frac{r_k}{g_k})$ is an irreducible binomial (that is, $\frac{a_n}{g_k}$ and $\frac{r_k}{g_k}$ are relatively prime) it follows that $\frac{g_1 g_2\ \cdots\ g_n}{a_n^{(n-1)}}$ is a positive integer, otherwise the polynomial $p(x)$ will have some non-integer rational coefficients, contrary to our assumption. On the other hand, since the only common factor between all coefficients of $p(x)$ is 1 we conclude that $\frac{g_1 g_2\ \cdots\ g_n}{a_n^{(n-1)}}$ can't be greater than 1. Therefore $\frac{g_1 g_2\ \cdots\ g_n}{a_n^{(n-1)}} = 1$ and proof of the algorithm is complete.

Teaching on the Edge

by Mike Physick

Mike is a B.C. bread teacher now teaching in inner-city Los Angeles.

It's another sunny Monday morning in Los Angeles and the school day begins as four large yellow school buses wait outside the gated campus. As one of L.A. Unified's continuation schools, Foster CDS accepts many students who have been expelled from regular high schools for everything from attendance issues to severe behavior problems. Students lie with their heads against the vinyl seats, catching as many lost minutes as possible from their event-filled weekends. Suddenly it's time. The call of the driver wakes them up to form a line at the security check. Each student is searched thoroughly and then "wanded" for concealed metal objects. To enter there must be no food or drink, no sharp metal object, and most importantly, no gang colours or signs visible to the eye. Each student must have one HB pencil and paper in a binder for taking notes.

Today is Juan's first day of school in three years. After being kicked out of his home school in the 6th grade for threatening to kill his teacher he decided to "take a year off." Juan's two older brothers who are members of a well know gang in Wilmington were more than happy to find him "jobs" to do. Consequently, Juan spent most of his last year in "Los Padrinos" or "LP" as they call it, which is the juvenile detention facility in the city of Los Angeles. After release from LP, Juan's G.P.S. ankle bracelet keeps him on house arrest and hopefully out of trouble. This might be just what he needs to get his life back on track. As Juan passes through the security check his mechanical pencil is taken away. He is told by another student that mechanical pencils were banned from the school three years ago when "some fool" was stabbed in the neck – "that fool's blood was everywhere!"

Breakfast is served first thing in morning to everyone and Juan looks for a place to sit and eat his breakfast burrito. The student body, which is essentially one quarter African American three quarters Latino, are divided as such. Within these groups many students sit according to the cities that they come from. Not everyone is involved with gangs, but those that are; pay close attention to where Juan chooses to sit. Juan finds a familiar face and sits to eat his breakfast.

It's been three weeks since house arrest and Juan's body is having difficulty adjusting to being sober. Headaches in the morning and an unsettled stomach are only a couple of the by-products. Once a week he must meet his parole officer for a drug test as ordered by the court. Many students at the school have severe drug and alcohol addictions, lucky for Juan; he has been forced into rehab.

First bell rings and it's off to class. Students rise out of their seats and a few clash on the way to period one. As I greet each student on the way in, I see Juan and offer him my regular cheerful "good morning" – no response. Those that I have taught for more than a couple of months slowly begin to respond with a "good morning Mr. Physick." Many students choose to return the greeting in a way that has meaning to them such as: "Yo

P, wazz-up" or "what's crackin' hommie." Some reach out a fist, so I return the gesture with a friendly "pound." In my first two months of teaching at Foster I quickly learned how important the first 5 minutes of class are – if I do not greet every student individually and direct them to their seat, I can consider the rest of class a write off.

Juan takes his regular seat according to the seating plan and puts his head down on the table instantly. Somehow, amidst everything that Juan is trying to deal with in his personal life, I have to teach him enough mathematics to pass the high school exit exam so he can graduate. The intended lesson for today is on solving simple equations. How can Juan be motivated to do mathematics when his most basic needs (physiological and safety) are not being met? Juan has very little stability in his life. He, and many of his classmates, because of classes missed, lack of family support, the effects of drug use, and lack of self esteem are stuck at roughly a grade four or five mathematics level. Some in the class are working at a high school level and many are in between the two extremes. That said, creating lessons that fit into every student's "zone of proximal development" is challenging, if not impossible. Also, when more than half of the students attend class less than three quarters of the time, designing a "set" of lessons that build from one to the next almost seems counter-productive. Instead, I am forced to teach each lesson as if it is its own entity and in my planning I often find myself thinking "now how would I teach this concept to someone who has never seen mathematics before?" As the frustration subsides, I begin to realize that in looking for the answer to this question I am learning more and more every day about what it means to help students "construct" ideas for themselves.

Class begins and Juan is getting in a comfortable position for a nap when I tap him on the shoulder and give him a problem on a piece of paper to try. I ask him to see if he can find a number that makes the equation true – just by guessing and checking. He crumples the piece of paper and replies "F-you, I F-ing hate math." I let Juan be for the time being, knowing that this only marks the beginning of my efforts with him. Juan is not the first here to express hatred for the subject of mathematics and he certainly will not be the last. In fact, as I reflect back on my experiences teaching at a "normal" high school in Port Coquitlam, I am sure that many students felt the same as Juan but, out of respect, expressed themselves differently. Here in this small Los Angeles continuation school, no emotion is held back – here you always know where your students stand and where you stand with your students. You can't take anything personally and at the same time you must be both fair and real in all of your interactions with students.

Directing students towards the task at hand is difficult today because of last night's shooting. Stephanie explains to the class that the girl shot at the football game in the stands at Carson High was her cousin. She says that she is going to be all right because "it only hit her in the leg." With this, three other students begin sharing their own stories of "flesh wounds" that they "earned" over the summer. Omar shares with the class that over the summer, his Aunt was killed in a "drive by." This type of street violence is real to many of these students - some keep pictures of loved ones who have passed on in the front sleeve of their folder. When serious issues like these present themselves in class it is often the case that the mathematics has to take the sidelines and we take some time in class to discuss what has happened. However, today it seems like the class, for the most part, is ready to move on.

As students find solutions to their simple equations (some by trial and error, some by equation solving techniques, some by simply guessing) I ask them to find two other students in the class with the same solution. When orchestrating activities that involve movement and that lead to group work it is critical to understand the dynamics of the students in each class. In this type of a setting if the wrong two students are paired at the wrong time the result could end up being violent. I learned this lesson the hard way in my first week when two rival gang members were put together in a group for an activity and a fight broke out in the classroom. Since that experience, I spend a lot of time considering how students could be grouped in each class for such activities. On the one hand you have to be careful and respectful of the divisions that exist, on the other hand, you want to do small things to encourage acceptance and social growth.

I return to Juan and give him a hint. I think he is surprised that I'm helping him at all after what was said minutes ago. I tell him I'm proud of him for making it to school today and empathize with him about how difficult it must be to be back after three years on the streets and in LP. He nods. I point to the piece of paper and tell him "you know … you *can* do this." I say the problem to him in words "two times a number plus one is eleven." I say it two more times. I think now he knows I'm not going anywhere. When I see that he is thinking of what it could be I know I have him – at least for today. He quickly says "five," followed by "this sh-t is easy." Juan is soon found by two other students with "five" as the answer and they compare their results.

To teach here at Porter CDS you have to be patient, deliberate in your pedagogy, and real to your students. You need the patience to see through the sometimes verbally abusive comments to the abused child crying for fair and firm boundaries. Your lessons must be planned so that, whether it's a student who has high ability in mathematics or is walking in to a math class for the first time in three years, he or she can participate on some level and experience some form of success – maybe for the first time. Most importantly, you must be sincere and real in your interactions with students so that they begin to feel the positive effects of a safe and fair learning environment – and when one student like Juan takes a step in a positive direction, you are motivated to persist in your efforts.

Curve Sketching Part II: A Closer Look at Critical Points

by Duncan McDougall

Part I of this article appeared in the Fall 2004 issue of *Vector*. Duncan McDougall is currently Director/Proprietor of TutorFind Learning Centre in Victoria BC.

In a follow up to his 2004 article entitled "Curve Sketching: An Eight-Step Menu for Polynomial and Rational Functions" which he developed as a request from students at Royal Roads University, Duncan McDougall noticed that students who liked his approach suggested that it was too bad that a menu did not exist for the beginner curve-sketcher. This article is the result of the effort to rectify that.

If we explore the analysis and graph of functions which are neither polynomial nor rational, we quickly realize the importance of a comprehensive definition for critical points, the various types of critical points, their proper calculation, and their order in the analysis of the function to be graphed. I believe that identifying these values at the earliest part of the analysis is crucial to successful sketching because it eliminates confusion. There is so much relevant information to analyze and accumulate at one time that a set procedure to follow facilitates the process. With this in mind, let us establish the following definitions: 1. Critical Points: The set of all values of x for which the first derivative is either zero or undefined. 2. Extrema or Maxima/Minima: The values of x for which the numerator of the derivative expression is zero or for which the derivative of the polynomial is zero. 3. Vertical Asymptote: The values of x for which the denominator of the given rational expression equals zero, and 4. Vertical Tangents: The values of x for which the given expression $f(x)$ is defined, but the derivative expression is not defined. Let us apply these definitions to the following examples beginning with a polynomial: $f(x) = x^3 + x^2 - 16x - 16$. The critical points for the derivative polynomial $f'(x) = 3x^2 + 2x - 16$ are $x = -8/3$ and $x = 2$. These are extrema only, as there are no vertical asymptotes or vertical tangents for polynomial functions. Rational expressions such as $f(x) = \dfrac{x-2}{x+4}$ (1) or $f(x) = \dfrac{x^2+3}{x-1}$ (2) are more interesting because they require more thought. The quick identification of $x = -4$ in (1) takes that value out of consideration for any other category other than vertical asymptote. However, the derivative of $f(x) = \dfrac{x-2}{x+4}$ is $f'(x) = \dfrac{6}{(x+4)^2}$, and this yields no extrema. Since $x = -4$ has already been labeled as a vertical asymptote, we quickly move on to the behavior of the given curve. Here, we include all critical points including all vertical asymptotes and all vertical tangents, in essence, any value of x which causes an interruption of the number line. And, since there are no extrema, the discussion is limited to activity before and after $x = 4$. For $f(x) = \frac{x^2+3}{x-1}$, we have a vertical asymptote at $x=1$ (as well as a diagonal asymptote at $y = x+1$), and extrema at $f'(x) = \frac{(x+1)(x-3)}{(x-1)^2}$. Here we equate both $(x+1)$ and $(x-3)$ to zero for extrema and leave $x=1$ alone, as it has already been designated a vertical asymptote.

Now consider the non-rational function $f(x) = x\sqrt{x+3}$, $x \geq -3$. Note that $f(x)$ is defined for $f(-3)$. However, the first derivative $f'(x) = \frac{3(x+2)}{2(x+3)^{1/2}}$ yields $x = -3$ as a vertical tangent because $f'(-3)$ is undefined. The x-value of -2 becomes a candidate for extrema because it came from the numerator. In further derivatives, we would not be concerned with $x = -3$; whereas had it not been classified, the second derivative $f''(x) = \frac{3(x+4)}{4(x+3)^{3/2}}$ would be more difficult to assess. Instead $x = -4$ becomes our only point of inflection without concern for $x = -3$.

In our next example, $f(x) = x^2 + \ln x$, $x > 0$, the derivative is $f'(x) = 2x + \frac{1}{x} = \frac{2x^2+1}{x}$. Here $x = 0$ would be a vertical asymptote and there would be no candidates for extrema because $2x^2 + 1 \neq 0$. In terms of $f''(x) = 2 - \frac{1}{x^2} = \frac{2x^2-1}{x^2}$, $x = 0$ remains a vertical asymptote, thereby not confusing the analysis. Thus we need only concentrate on $2x^2 - 1 = 0$ or $x = \pm\frac{\sqrt{2}}{2}$ for our points of inflection.

This clarification on this important topic of critical points is designed to ease the difficulty with curve sketching by allowing the reader to account for every value of x incurred and to use it to its best advantage. I truly believe that an ordered sequence of steps helps the beginner with what can be a complicated procedure. And once the discussion of critical points has taken place, the rest of the analysis is much easier.

A Position Paper on the Usefulness of *Academic* Mathematics

by Ed Atwater

This article first appeared in *Vector* in May 1979, volume 20, number 3.

At a recent staff meeting, there was some discussion about the value of the present education system for the "average student." The negative side of the argument went something like this: Since only 5% of the students will be going on to university, why do the other 95% have to take the academic material designed for those few?

Being an optimist and a pragmatist (and a few other things), I have thought about where math fits into this controversy. I feel (strongly enough to write this position paper) that "academic math" is tailor-made for both groups. The following are arguments to support this tenet, some only sketchily presented, some more elaborately defended, all admittedly shaky, and none intended with malice to the reader's philosophical orientation. (The list is by no means exhaustive; please add your own reasons.)

1. Since the teacher does not know who is going to need math later in life, it is better to prepare each as if he/she were the one. If those who never use it are getting something as valuable as, or more valuable than, what they would have gotten had they not taken "academic math," I think my position is tenable.
2. The curriculum guide for the new math courses prescribes more "real-world" math in all courses than in the past. Problem areas related to commercial, technical, and scientific fields can be taken for granted as valuable for something. Theses areas may be used as sources of problems to use some of the more abstract concepts learned as "algebra" or treated as entities unto themselves.
3. One thing found constantly in academic math is an algorithm, a specific, detailed sequential set of steps to follow. The Income Tax Guide and a manufacturer's instructions for putting a toy together are "real world" examples of algorithms. Now, one could teach only specific, "useful" algorithms, and in some classes, that is done. But they are subject to change and obsolescence. By teaching more algorithms in a more general, abstract form, some theorists claim the student is better able to deal with new or different algorithms as they are encountered later in life. I am one of those theorists, and my teaching emphasis is more often on the steps than on the answer. I give mostly open-book exams in Grade 8 and 9, trying to get student familiar with the skills of reading and following instructions and examples. For example, geometric constructions as a means to get students to follow a detailed, sequential set of instructions- to read and understand- are valuable exercises, regardless of the student's eventual path in life.
4. Math teaches logical, analytical, and critical thinking. If you are a "pro-transfer-

person," you may accept that as math is necessarily all these things, it is possible that in learning to do math, one may be learning thinking skills, techniques, patterns, tricks, etc., that may be functional in other aspects of life. If you are an "anti-transfer-person," you may dismiss this point altogether.

5. Math is part of our heritage, our present, and our future. Every intelligent person should know a little about math beyond arithmetic in an increasingly technical world. To be totally ignorant of abstract mathematical processes is a travesty; one still may not be able to do much, but one is aware that there is rhyme and reason behind all the "technical stuff" around us. Without some concept of the things taught in "academic math," one is left to assume it is magic.

6. I use math as a medium to teach a plethora of "old-fashioned" concepts. I will couch them in cause-effect relationships, which all teachers are dealing with in the "hidden curriculum."

    - Cause: not doing the assigned work. Effect: you come up empty-handed when called to account. I rarely check the assignment by walking around looking at notebooks; I give frequent little quizzes, which require knowledge of they key concepts practiced on the assignments. Those caught empty-handed (empty-headed?) are "encouraged to attend an extra session" to practice those concepts. This may or may not change many slacker's effort, but if the above cause-effect is learned, when it becomes important to that person not to come up empty-handed, he/she will know that planning and preparation prevent poor performance, whether in university or on the job.
    - Cause: lack of organization. Effect: inability to find information being found by others around you. I provide guidelines and directions on the organization and use of notebooks. I have designed my own courses so that the well organized notebook is valuable to the student; it helps him/her get a better mark and makes life easier along the way. I allow the use of a notebook on the test; only if it is organized in an understandable manner will it be in the notebook; we look it up, and if it is not there, we put it there. If I can teach the student something about a functional source of information, whether he/she goes on the university or not, he/she has learned a valuable skill.
    - Cause: not asking questions. Effect: being caught empty-handed when called to account. I go looking for the "unasked" question on a quiz. Often enough, a few instances of this are enough to convince some students that it is worthwhile to ask questions. The learned relationship between asking the right question and avoiding being caught empty-handed might come in handy some day, regardless of what a student does after grade 12.
    - Cause: lack of self-discipline. Effect: poor performance when given free reign. In Grade 8, substantial external control is usually required to get any work done. But, through the year and in Grade 9, I intermittently relax that control in areas that are less crucial to subsequent performance. I then point out explicitly and emphatically what I have done and what has happened when a drop in marks occurs under these circumstances. The grade 10 student is entering the world of the young adult; if he/she has never had to exercise any self-discipline or suffer the consequences of not being self-disciplined, full adulthood, with all its responsibilities and with no external control in many areas, can be very unpleasant. By seeing the relationship in

school between lack of self-discipline and poor performance, perhaps a student will not have to learn the lesson the hard way later on, in university, or in the working world.

- Cause: lack of proper equipment to do the job. Effect: wasted time and an unfinished job. I think more than one student would be rich if they had a nickel for every time they heard, "I can't go around looking after you all your life!" But then we loan them books, because they forgot theirs, or loan pencils because they lost theirs, or extend the deadline so they can get their assignments done, or… Not too many steadily employed people show up consistently without the necessary tools; I feel obligated to teach some students that there are instances where someone will not carry Kleenex around for them.
- Cause: carelessness. Effect: a needless error. The last point, like all the others, is learned in a setting where learning the relationship the hard way will usually have no long-range deleterious effect. Because math is not intrinsically motivational, like a lot of life's little chores, the good habits I am attempting to instill are learned in a setting not unlike those where the habits will be useful later in life. If good habits are not formed, math is not learned, but the consequences do not ruin one's life. Learning the same lessons at university, on the job, in a car, or in a marriage could "ruin your life."

With a little help, I have learned I teach students, not math. If a "little" math gets picked up along with teaching those students, then I really feel good. I would be the last to teach academic math to every student for the sake of academic math. But as a medium to teach all those other things, I find math perfect. I think what I am doing is valuable to every student in every period every day. And even though only a couple may ever use what I am teaching, the other taking "academic math" are getting something valuable far beyond glorified arithmetic.

Mathmagic VII: The Final Frontier

by Al Sarna

This article is based on the presentation given at the Northwest Mathematics Conference in the Fall of 2009. Al Sarna is currently the Head of mathematics at Kitsilano Secondary School.

Another Impossible Revelation

The spectator has 4 slips of paper. They choose any number, flip the slip and write the next consecutive number on the back. On the second slip goes the next consecutive number and so on until the spectator has 4 slips of paper with 8 numbers.

1. The spectator places the slips of paper face up with the higher numbers showing.
2. They flip one slip and add the numbers.
3. They tell the mathemagician their *original starting* number and he tells them their sum!

Solution

Suppose the starting number is 9. The mathemagician simply goes $4x + 15$ where $x = 9$. The sum is 51.

Let the 4 slips of paper be as follows where x is the starting (and top) number. Below is the arrangement. The numbers the spectator will have face up are in the bottom row.

x	$x+2$	$x+4$	$x+6$
$x+1$	$x+3$	$x+5$	$x+7$

Before the flips the sum of the bottom numbers is $4x + 16$. No matter which one is flipped the sum decreases by 1 to give $4x + 15$.

- This generalizes nicely to any number of slips of paper with any constant d. In other words, this could be adapted to use with arithmetic sequences and series.
- For instance, let there still be 4 slips of paper but with a constant difference of d as follows:

x	$x+2d$	$x+4d$	$x+6d$
$x+d$	$x+3d$	$x+5d$	$x+7d$

Before the flip the sum is $4x + 16d$. No matter which one is flipped the sum decreases by d to give $4x + 15d$. For example, if $x = 5$ and $d = 4$ the sum will be $4(5) + 15(4) = 80$.

- The next step in the generalization is to start with x (or a, as is commonly used for sequences and series), let the difference be d, and let the number of slips be n.
- Depending on how far you want to take this it can be used anywhere from grade 8 to grade 12!

IMPOSSIBLE 37

A spectator selects any 3 – digit number whose digits are all the same. They add up the 3 digits and divide this sum into the original number. The mathemagician reveals the answer of 37.

1. Suppose the number is 444.
2. The sum of the digits is 12.
3. 444 divided by 12 is 37!

Solution

If the digits are all the same the 3 – digit number will be $N = 100x + 10x + x$. The sum of the digits is $3x$. The quotient is $\frac{100x+10x+x}{3x} = \frac{111x}{3x} = 37$.

- This is a nice little exercise for grade 8 or 9 students.
- This does not generalize the way one might think. For instance, if you apply this to a 4 – digit number you would have to divide the number by 11 times the common digit to get a 'magic' result of 101.
- For a 5 – digit number you have a problem!

THE SPELLING CARDS

The spectator picks, unknown to the mathemagician, from 1 to 12 cards and puts them in their pocket. They also look at the card at that position (number) in the remainder of the deck. They pick the name of a famous person (or someone in the class!) whose name has more than 12 letters. The mathemagician demonstrates how to place the cards as they spell the name, then they replace their pile to the top of the deck, spell the name, and the next card is the one they looked at!

1. Suppose the spectator pockets 7 cards (from 1 to 12). He then looks at the 7^{th} card from the top of the deck and remembers this card.
2. They pick a name with more than 12 letters; say George Clooney.
3. The mathemagician demonstrates the spelling by placing the cards, one at a time, face down on the table. These cards are then placed on the top of the deck.
4. Next the mathemagician has the spectator place his packet on the top of the deck stressing that he has no idea of how many cards were taken.
5. The spectator now deals the cards, one at a time, and face down spelling G-E-O-R-G-E-C-L-O-O-N-E-Y.
6. The next card will be the noted/chosen card!

Solution

Let the spectator choose x cards such that $1 \leq x \leq 12$ and let the name have y letters where $y > 12$. The spectator has x cards in his pocket and looks at the x^{th} card from the top.

$\frac{x}{y-x}$	Now the mathemagician spells out the name reversing the cards as the diagram at the right shows.	$\frac{y-x}{x}$

Now the spectator replaces his packet of x cards on top (of the pile at the right). This gives us $x + (y - x) = y$. Therefore, after spelling out the y letters the next card is the noted card.

Note: if you feel you might have trouble getting a name with 12 letters simply restrict x to $1 \leq x \leq 10$, or to whatever number works for you!

THE REAPPEARING 10890!

The student picks a 4 – digit number, performs a few calculations, and always arrives at 10890. This is an extension of the old ‘1089’ magic trick. The proof is somewhat more complicated and you may want to use this with grade 10 or 11s.

1. The student picks any 4 – digit number such that the digits decrease in value from left to right.
2. Then they reverse the digits and subtract this new (smaller) number from the original number.
3. Next they reverse the digits of this difference and add it to the answer from step 2.
4. They will always get 10890!

Example

Pick 9743. Reverse the digits to get 3473. Subtract to get 6264. Reverse the digits to get 4626. Add to 6264 to get 10890.

```
  9743
 -3479
  6264
 +4626
 10890
```

Solution

Let the number be $N = 1000x + 100h + 10t + u$. Next we need to reverse the digits and subtract:

$$\begin{array}{l} \quad 1000x + 100h + 10t + u \\ -\underline{\left(1000u + 100t + 10h + x\right)} \end{array}$$

The problem here is that x is at least 3 more than u so we need to borrow 10 from t as follows:

$$\begin{array}{l} \quad 1000x + 100h + 10\left(t-1\right) + \left(u + \underline{10}\right) \\ -\underline{\left(1000u + 100t + \quad 10h + \qquad x\right)} \end{array}$$

The next problem is that since $h > t$ then $h > \left(t-1\right)$ so we need to borrow 100 from h as follows:

$$\begin{array}{l} \quad 1000x + 100\left(h-1\right) + 10\left(t-1+\underline{10}\right) + \left(u + \underline{10}\right) \\ -\underline{\left(1000u + \quad 100t + \qquad 10h + \qquad x\right)} \end{array}$$

Now we know that $h > t$ by at least 1 so $\left(h-1\right) \geq t$ so we won't need to borrow from x. Our final calculations are as follows:

$$\begin{array}{lllll} \quad 1000x + & 100\left(h-1\right) + & 10\left(t-1+\underline{10}\right) + & \left(u+\underline{10}\right) \\ -\left(1000u + & 100t + & 10h + & x\right) \\ \hline 1000\left(x-u\right) + & 100\left(h-1-t\right) + & 10\left(t-h+9\right) + & \left(u+10-x\right) \\ +1000\left(u+10-x\right) + & 100\left(t-h+9\right) + & 10\left(h-1-t\right) + & \left(x-u\right) \\ \hline 1000\left(10\right) \quad + & 100\left(8\right) & + \ 10\left(8\right) & + \quad 10 \end{array}$$

$= 10\,890$

REVEALING THE CROSSED OUT DIGIT (AGAIN)

This is yet another variation on the theme of the old 'casting out nines' topic that used to be taught.

1. The spectator picks a number with 3 or more digits (with no zeroes), adds the digits together and subtracts this sum from his original number.
2. Then the spectator scrambles the digits in this answer and adds 25.
3. Finally, cross out a non – zero digit, subtract 7, add the remaining digits and then subtract 7 from this sum. Reveal this number.
4. The mathemagician reveals the crossed out digit by subtracting this sum from the next higher multiple of 9.

Example

- Let the number be — 58361
- Sum the digits — $5+8+3+6+1=23$
- Subtract this from the original number — $58361-23=58338$
- Scramble the digits — 35838
- Add 25 — $35838+25=35863$
- Cross out a (non – zero) digit — $3\not{5}863$
- Add the remaining digits — $3+8+6+3=20$
- Subtract 7 — $20-7=13$
- Crossed out digit is — $18-13=5$

If you are familiar with casting out nines simply reduce the last answer (13) to a single digit by adding $(1+3=4)$ and adding whatever is necessary to get to 9 (5).

Solution

Consider a general 4 – digit number $1000x+100h+10t+u$ (any number of digits can be used). If you subtract the sum of the digits you have

$$1000x+100h+10t+u-(x+h+t+u)=999x+99h+9t.$$

Notice that this result is divisible by 9.

If a number is divisible by 9 then the sum of its digits is divisible by 9 as the following rewriting of our original numbers shows:

$$1000x+100h+10t+u=999x+99h+9t+(x+h+t+u).$$

From here, scrambling the digits has no effect on the divisibility.

Now let's add 25 and cross out the h (for example) to give us $999x+\cancel{99h}+9t+25$. Now add the remaining digits together; $x+t+25$.

Finally, subtract 7 to get $x+t+25-7=x+t+18$.

When we subtracted the sum of the digits the number had to be divisible by 9. All the adding 25 and subtracting 7 did was making sure we kept the result divisible by 9. We could just as easily have said add 38 and then, somewhere else subtract 2 to get 36!

The reason I eliminated zeroes from the number is because both 0 and 9 are divisible by 9. For those who have done casting out nines a more mathematical reason is that both 0 and 9 leave the same remainder when divided by 9.

1. The mathemagician reverses 20 cards (turns face down cards face up) anywhere in the deck in full view of the spectator. The spectator thoroughly shuffles the cards without reversing their orientation (face up or face down).
2. The spectator turns away and counts off 20 cards from the top of the deck and hands this pile to the mathemagician (preferably behind his back).
3. The mathemagician stresses that neither of them can know how many cards are face up, but that it is likely less than the number of face up cards in their (the spectator's) packet of 32 cards.
4. The mathemagician reverses some cards and it turns out that they both have the same number of face up cards!

Solution

All the mathemagician has to do is reverse the packet of cards he is handed by the spectator! Think of it this way; let the 20 face up cards be distributed between the two piles as follows (think of reading across and not down):

(The pile of 20)	(The pile of 32)
x Face Up	$20-x$ Face Up
$20-x$ Face Down	

Now it is readily seen that flipping the pile of 20 will give us the $20-x$ cards face up!

This is a nice variation on the old 'water in the wine' and 'wine in the water' problem. It goes something like this; suppose you have 1 beaker of wine and 1 beaker of water. If you transfer 1 ml of water into the wine beaker and then 1 ml of the wine and water mixture into the water beaker is there more water in the wine or wine in the water? The answer is that there is exactly the same amount of wine in the water as there is water in the wine. In fact, it doesn't matter if the beakers contain the same amount or how many transfers are done! The only thing that is important is that they finish up with the same volume they started with.

Think of it this way. If x amount of wine is missing from the water beaker then the corresponding volume that it occupied must now be filled with the water. You could actually use numbers and use the problem at the grade 10 or 11 level if you wanted to have them work it out.

FINALLY

These are just a few of the many mathemagical 'tricks' you can use to spice up a math class or introduce a new lesson. Many more can be found in the books listed in the bibliography or by doing a search on the internet for things like "Math Magic", etc. Relax, enjoy, and may the force stay with you.

Bibliography

Fulves, K. (1983). *Self-working Number Magic.* New York, NY: Dover Publications.

Fulves, K. (1984). *More Self-working Card Tricks.* New York, NY: Dover Publications.

Gardner, M. (1956). *Mathematics Magic and Mystery.* New York, NY: Dover Publications.

Gardner, M. (1975). *Mathematical Puzzles and Diversion.* Pelican Books.

Mira, J. (1971). *Mathematical Teasers.* Barnes and Noble Imports.

Simon, W. (1964). *Mathematical Magic.* New York, NY: Charles Scribner's Sons.

Yawin, R. (1977). *Math Puzzles and Oddities*. Grosset & Dunlap.

Teaching Mathematics as Storytelling

Rina Zazkis & Peter Liljedahl

ISBN 978-90-8790-733-4
Sense Publishers 2009
Paperback, 138 pages

Reviewed by Darien Allan

Many might think that storytelling in mathematics is used solely to provide a welcome distraction from the oft-perceived drudgery of traditional mathematics. In their book, "Teaching Mathematics as Storytelling", Rina Zazkis and Peter Liljedahl successfully endeavour to convince the reader otherwise. They believe that storytelling provides multiple purposes and benefits to both students and teachers when teaching mathematics.

As written by Zazkis and Liljedahl, the reason for storytelling is to "make mathematics more enjoyable and more memorable" and to create an engaging and imaginative environment that promotes thinking and understanding. Throughout the book the authors provide helpful advice and many examples intended to educate and motivate the reader to use stories in the mathematics classroom. So although the writing level is accessible to almost any reader, the content is most relevant for those who teach mathematics.

make mathematics more enjoyable and more memorable

Chapter 1 begins as Zazkis and Liljedahl might envision any mathematics class beginning: with a story. This serves to pique the reader's interest and is followed by an explanation of what a story is, its purpose in the classroom, and a description of different kinds of stories. In the second chapter the authors provide a comprehensive description of the elements that are important to making a good story, using examples to demonstrate the effects of each. The final introductory chapter describes the processes and important considerations in the delivery of the story to the intended audience.

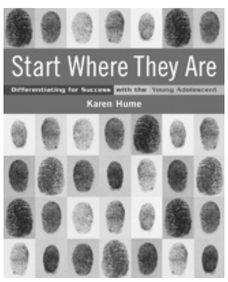
Chapters four through nine discuss the different purposes that stories have in the classroom. For example, stories can be used to: set a frame or a background (4), accompany or intertwine (5), introduce (6), explain (7), ask a question (8), and tell a joke (9). Each chapter provides a detailed explanation of the purpose of the story and gives at least one example of that type of story. The authors then dissect the story and discuss the key elements inherent within, thus providing storytelling fundamentals to the reader.

The last three chapters give examples and advice for readers to create their own stories, describe how one teacher in particular successfully used stories in his classroom, and include ideas for adapting current stories for classroom usage.

While reading this book one cannot help imagining how stories could be used in one's own classroom. The authors provide many ideas for how these stories can be used but do not provide "ready-made" stories for reader use. Readers are encouraged to make their own stories because stories must be adapted for both the teller and the audience to be most effective.

This book is both motivational and a great resource. Teachers and others will find it a valuable and inspiring addition to their library.

Start Where They Are: Differentiating for Success with the Young Adolescent



Karen Hume

ISBN 978-01-3206-913-7
Pearson Education 2007
Paperback, 288 pages

Reviewed by Karin Paterson

Sometimes, as educators, we want the “silver bullet.” Karen Hume’s book, *Start Where They Are: Differentiating for Success with the Young Adolescent* is just that … and it’s Canadian.

Hume carefully examines and explains the elements, conditions and pre-conditions of her framework for differentiating instruction, and she captures it in a single graphic organizer. The real strength of this book is Hume’s knowledgeable and encouraging voice that affirms that she understands us as teachers. This means the read is informative, engaging, highly interactive, and very practical. *Start Where They Are: Differentiating for Success with the Young Adolescent* is a handbook that you will return to again and again. Hume differentiates for her readers through an approachable layout of theory, practical suggestions, cartoons, references, and motivating quotes. Each chapter starts with a pre-reading activity, contains during-reading reflective tasks, and displays icons that direct you to the accompanying CD-ROM of blackline masters. Hume synthesizes such classic knowledge as Bloom’s Taxonomy, Gardner’s Multiple Intelligences, and Maslow’s Hierarchy of Needs with current understandings in the areas of brain research and assessment. She understands that as educators we do not need just another definition. We need to be able to make connections that have us asking ourselves, “How is this different from what I am already doing?” Hume makes it very clear that differentiating instruction is **<u>not</u>** juggling 30 different Individual Education Plans in one classroom. Rather, it is caring, thoughtful and strategic planning that provides students with a contained range of options that meet their learning needs.

> *How is this different from what I am already doing?*

This book crosses all subject areas. So for all teachers who say “Yes, but how do I apply this in math?” Hume provides some specific math examples but primarily provides “In Your Role” general applications you can engage in with your math department or learning team members. Since teaching is about learning and a balance between teacher, student and curriculum, this book reminds us that we teach students not subjects.

Hume inspires us to put our students first, to believe that they can and are willing to learn, to view teaching as an honourable calling, and to remember that the students are counting on us.

Outstanding Teachers of the Year – 2009

by David Van Bergeyk, BCAMT President

NEW TEACHER OF THE YEAR

– DANNY YOUNG –

One of Danny Young's nominators confesses that when she first met Danny a number of years ago, she assumed him to be an experienced teacher based on the insight and knowledge of mathematics teaching and learning he displayed. She was surprised to learn he was still a student teacher! Colleagues and students alike all attest to Danny's impressively deep understanding of issues related to mathematics teaching and learning. He is praised for employing teaching strategies that address multiple learning styles, and for his ability to create a dynamic and supportive classroom environment while still demanding the best from his students. In just a few short years he has developed a reputation as a dedicated and supportive professional among colleagues, and has become a favourite and much-sought-after teacher among students.

It seems enthusiasm is a defining feature of Danny's work. His enthusiasm for mathematics is infectious to his students, and his enthusiasm for teaching seems to catch with his colleagues. He is an enthusiastic user of technological innovations in teaching, which have made his lessons appealing and understandable to students and also support learners by making resources more widely accessible. Furthermore, Danny is now enthusiastically willing to share his growing expertise with the broader community by presenting at conferences. We enthusiastically congratulate Danny on his work in the classroom, and encourage him to continue to build on and share his early success.

SECONDARY TEACHER OF THE YEAR

– TRISHA WONG –

For many years, Trisha Wong has demonstrated excellence in teaching mathematics. Her knowledge of the mathematics curriculum is evident in the breadth of her teaching experience, and also in her work on district and ministry committees. She has worked with the Ministry of Education on marking provincial exams, evaluating resources, and creating curriculum support materials in the form of a Classroom Assessment Model. At the District level, Trisha has been a leader in curriculum implementation planning and developing assessment instruments for collecting district data.

While knowledgeable work in these endeavours is impressive, uppermost in our minds as we honour Trisha with this award is her outstanding classroom teaching. She regularly employs cooperative learning principles and empowers students as tutors to support learning in her classroom. Her class activities are enriched by creative project ideas, including the student use of technology to demonstrate learning. These projects are a part of Trisha's strong emphasis on formative assessment strategies, which support students with multiple opportunities to acquire and demonstrate their learning. In all, Trisha is also noted for her outstanding dedication to her students, making herself available to them on a drop-in basis and also running extra evening and weekend tutorials.

In addition to excellent teaching in her own classroom, Trisha's excellence extends to her impact on colleagues within her own department, her district, and beyond. Trisha has for a number of years contributed to professional growth in others by offering a number of seminars locally and at provincial conferences. Perhaps her most significant contribution in this regard is the work she has done to help others use tablet PC technology effectively in instruction. Trisha's expert use of technology obviously positively influences learning in her own classroom, and we commend her willingness to share her expertise and passion with others. We congratulate Trisha on winning this award and look forward to your continued contributions to excellence in mathematics education.

CALL FOR SUBMISSIONS

We are looking for quality submissions of the following:

- research reports
- literature reviews
- stories of teaching
- teacher resources
- numeracy tasks
- relevant website links
- interesting problems
- students' solutions to problems
- book reviews
- letter to the editor

Articles can be submitted by email to the editors listed on page 2. Authors should also include a short biographical statement of 40 words or less. Articles should be in a common word processing format such as Apple Works, Microsoft Works, Microsoft Word (Mac or Windows), etc. All diagrams should be in TIFF, GIF, JPEG, BMP, or PICT formats. Photographs should be of high quality to facilitate scanning. The editors reserve the right to edit for clarity, brevity, and grammar.

GRADES k-1

Charlie, Susan, and Amber get to share six cookies. However, Susan's mother has told her that she is only allowed to have one cookie. How do you share the cookies?

GRADES 2-3

When a bucket is full it holds exactly 5 1/2 litres. A jug holds 500 millilitres. How many full jugs of water will I need to fill the bucket?

GRADES 4-6

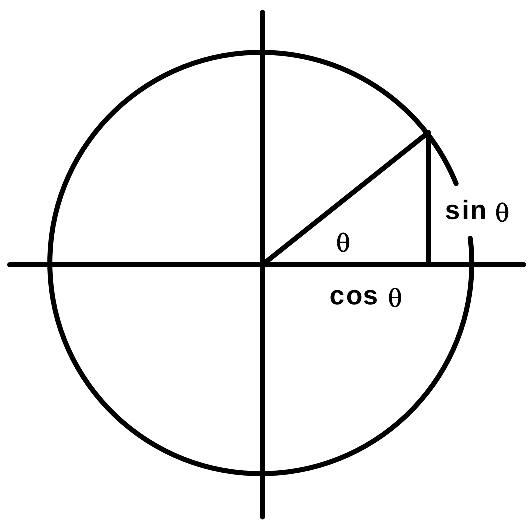
Roll five dice and then stack them on top of each other to make a tall tower. Now calculate the sum of the faces that are hidden from view. How can you do this quickly?

GRADES 7-9

A 600 pound pumpkin was entered in a contest. When it arrived it was 99% water. The pumpkin sat for days in the hot sun, lost some weight (water only), and is now 98.5% water. How much does it now weigh?

GRADES 10-12

Consider the following diagram of the unit circle with a standard position angle θ. The line segments for sin θ and cos θ have been labelled. Where are the line segments for tan θ, csc θ, sec θ, and cot θ?



CALCULUS

Consider the differentiable function:

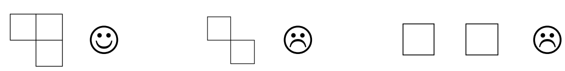
$$f(x) = \begin{cases} x^2 + 3, & x < 2 \\ mx + b, & x \geq 2 \end{cases}$$

Find m and b.

NUMERACY 5 TASK

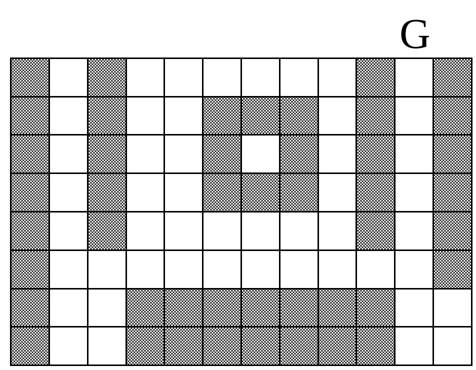
Your class is going to build a garden to grow plants to sell. The garden will be made up of planter boxes and paving stones to walk on. The garden must be wheel chair accessible so the paving stones are wide enough for wheel chairs. In order for the garden to grow well it must be weeded and watered regularly. To make sure that you can do this there are a few design rules to follow:

1. You must be able to walk beside each planter box on at least one side. This way you can take care of the plants in that box.
2. You are not allowed to step over any planter boxes. This is bad for the plants.
3. Paving stones must be connected along at least one side so that the path is wide enough for wheel chairs.



4. There must be a fence around the garden to keep animals out. The fence must have a gate. You can position the gate anywhere you want.

Last year's class designed the following garden with 46 planter boxes:



(shaded squares are planter boxes, white squares are paving stones.)

NUMERACY 8 TASK

The grade eight ski club is going to Grouse Mountain. Each person tried their best to raise money for their trip. Below is a chart that shows how much money each person raised. It also shows each student's individual cost based on whether they needed rentals or lessons.

Determine whether they have raised enough money for their trip. What would be a fair way to share the money that was fundraised among the people listed below? All of the money raised must be applied to the cost of the trip, and every person must go on the trip, even if it means that they may have to put in their own money to do it.

Name	Amount Raised	Rental Cost	Lift Ticket	Lesson Cost
Alex	75	20	40	40
Hilary	125	10	40	40
Danica	50	30	40	0
Kevin	10	40	40	40
Jane	25	0	40	0
Ramona	10	0	40	40
Terry	38	30	40	0
Steve	22	40	40	40
Sonia	200	20	40	0
Kate	60	25	40	0

Math Problems – grades 7,8,9

Use the headings under the ***Quick Links*** menu on the left to select problems by strand or learning outcomes.

http://www.galileo.org/math/MathProblems.html

Math Comics

An extensive collection from the library of Tim Barss.

http://courses.spectrum.sd61.bc.ca/teacherFolders/Barss/Cartoons/

Designing Backpacks

Columbia Sportswear Designer Chris Araujo combines innovation with design to create backpacks for one of the largest outdoor apparel companies in the world.

http://www.thefutureschannel.com/dockets/realworld/designing_backpacks/

Mathematics in the Movies

This is a collection of movie clips in which Mathematics appears.

http://www.math.harvard.edu/~knill/mathmovies/