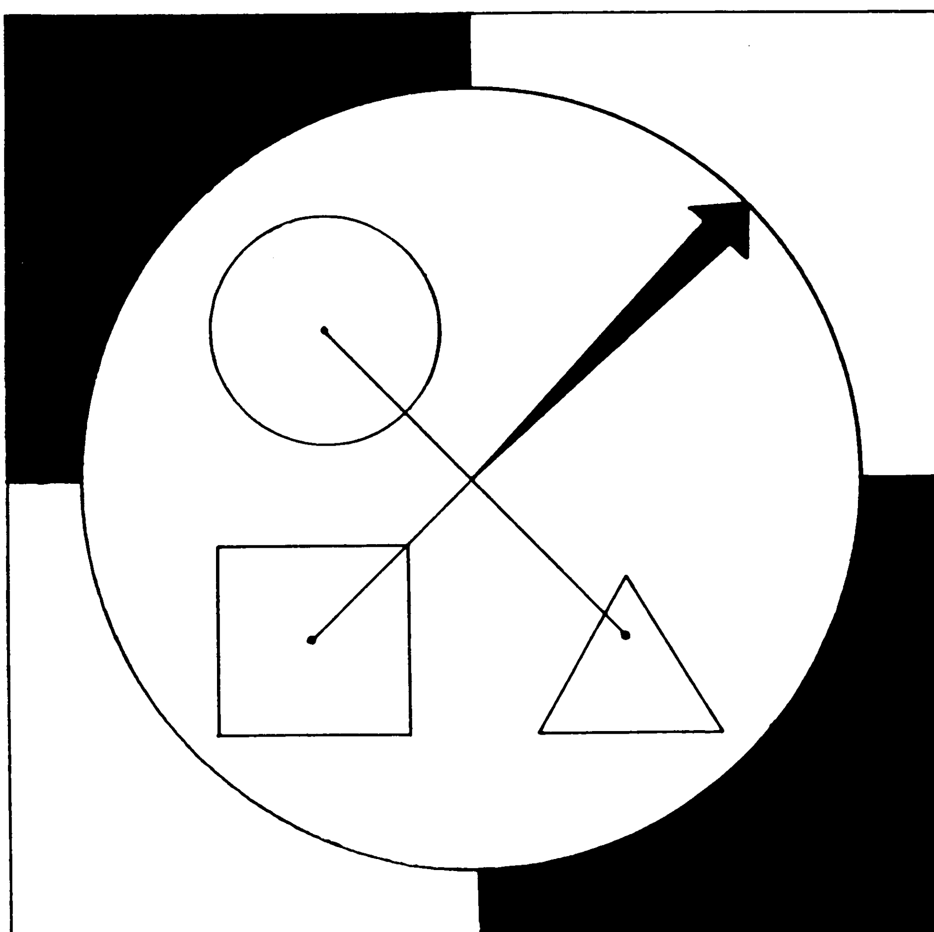
Vector

BRITISH COLUMBIA ASSOCIATION OF MATHEMATICS TEACHERS

NEWSLETTER



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The B. C. Association of Mathematics Teachers publishes Vector *(newsletter) and* Teaching Mathematics *(journal). Membership in the association is $4.00 a year. Any person interested in mathematics education in British Columbia is eligible for membership in the BCAMT. Journals may be purchased at a single copy rate of $1.50. Please direct enquiries to the Publications Chairman.*

AGM Coming Up!

Mike Baker, BCAMT president, has announced plans for the annual general meeting of the association, slated to be held in the BCTF Auditorium, 2235 Burrard Street, Vancouver, on Wednesday, April 5. A registration fee of $2 will be charged non-members of the BCAMT. There will be no charge for those whose memberships are up to date. Coffee and refreshments will be available.

The program will be as follows:

9:00 a.m. - 9:30 a.m.	Association Business
9:30 a.m. - 11:00 a.m.	Curriculum Revision -- a discussion with members of the Revision Committee on the philosophy, intent, and progress of the present revision (Elementary and Secondary).
11:00 a.m. - 12:30 p.m.	Effects of the Changed Requirements for High School Graduation

(a) After Graduation: What effects will a non-compulsory Math 11 have on job opportunities and on entrance to universities, technical schools and junior colleges.

(b) High School Mathematics: What effects could the changed regulations have on the present and future curriculum in both junior and senior secondaries.

Representatives from the various institutions, from the BCTF Curriculum Committee and from the BCAMT executive will present their views.

Mathematics In-service Teams

As was reported in the last issue of *Vector*, letters have been sent to all directors of instruction, department heads of mathematics, junior colleges and the universities in which assistance was sought to find competent individuals to staff mathematics in-service teams. The response thus far has been gratifying. We have offers of assistance from about thirty people, these having been recommended by approximately ten directors of instruction or mathematics department heads. We are in the process of contacting each of the volunteers. Once this has been done, a final list of resource people will be prepared for circulation to appropriate people.

If by chance you have not received a letter regarding our in-service program and by your position you should have, please contact me immediately. If you have as yet neglected to respond, may I urge you to do so immediately. Send your replies to:

A.J. (Sandy) Dawson
Faculty of Education
Simon Fraser University
Burnaby 2, B.C.

Prairie Beckons Math Teachers

Percy Cuttle, chairman, and Robert Kalapaca, program chairman, have extended an invitation to math teachers to attend '3 Big Days' at Saskatoon, August 23, 24 and 25, on the campus of the University of Saskatchewan. The conference is jointly sponsored by the National Council of Teachers of Mathematics and the Saskatchewan Mathematics Teachers' Society.

Not satisfied with building their own Blackstrap Mountain, the prairie people promise the most outstanding conference ever held on the prairies. A preliminary program report in the *SMTS Newsletter* displays a variety of topics and personnel which should prove very attractive. For further information write Dr. Cuttle at the Mathematics Department, University of Saskatchewan, Saskatoon.

When Is y a Function of x?

Dr. Hugh Thurston, Department of Mathematics, UBC.

Now that most textbooks give a clear definition of 'function,' this question should be easy to answer. However, to define 'function' is not quite the same thing as to define '...is a function of...'

Let us consider two possible definitions of the latter.

(I) A variable y is a function of a variable x if each value of x uniquely determines a corresponding value for y.

(II) A variable y is a function of a variable x if there is a function f for which $y = f(x)$.

The first definition can be found in many books, with various wordings. The second definition is an obvious definition of '...is a function of...' for anyone who has already defined 'function.'

Both definitions refer to variables, not to numbers: we never ask, 'Is 2 a function of 3?' What is meant by 'variable' in this context? Let us start by asking what exactly we mean when we say that the area of a circle is a function of its radius. We mean that there is a function f such that for any circle we choose, if its radius is ρ and its area α, then

$$\alpha = f(\rho).$$

(The function f is, of course, defined by $f(\xi) = \pi.\xi^2$ for every positive ξ.)

Notice carefully that one function has to serve for all circles. It is no use picking one circle (say with radius 2 and area 4π) and asking if 4π is a function of 2. In fact, if $r(C)$ denotes the radius of the circle C and $a(C)$ its area, we require

(i) $$a(C) = f(r(C)) \text{ for every circle } C.$$

It is now obvious that the 'variables' a and r are functions whose domain is the set of all circles. Thus equation (i) is equivalent to

(ii) $$a = f(r)$$

by the very meaning of equality between functions. (Here f(r) denotes a composite function, or 'function of a function.')

Fortunately, there is no need to ask which of our two definitions is the 'right' one; they are equivalent, and either will serve. The proof that they are equivalent is quite straightforward. The first step is to rewrite (I) in more mathematical language. A typical value of x is $x(\xi)$, where ξ is in the domain of x. The corresponding value of y is, of course, $y(\xi)$ and 'each value of x uniquely determines a corresponding value for y' means '$x(\xi) = x(\eta)$ implies $y(\xi) = y(\eta)$.'

If (II) holds, then $x(\xi) = x(\eta)$ implies that $y(\xi) = f(x(\xi)) = f(x(\eta)) = y(\eta)$, and so (I) holds.

Conversely, if (I) holds, we define f as follows. If α is any value of x, there is a ξ such that $\alpha = x(\xi)$. Property (I) assures us that $y(\xi)$ is uniquely determined by α, even though ξ itself may not be. Thus we can define $f(\alpha)$ to be the $y(\xi)$ just described. This defines a function f (whose domain is the set of values of x). Then $y(\xi) = f(\alpha) = f(x(\xi))$. This is true for every ξ in the domain of x, and so $y = f(x)$.

A practical way to see whether or not y is a function of x is from the graph of y against x. If no two points of this graph have the same x-coordinate, then y is a function of x.

We have taken it for granted that x and y have the same domain. Thus, to sum up, we have the following result.

Let x and y be functions with the same domain. Then y is a function of x if there is a function f such that $y = f(x)$. This will happen if and only if $y(\xi) = y(\eta)$ whenever $x(\xi) = x(\eta)$.

We can now ask some interesting questions. Let a particle move with velocity v and displacement x along a straight line. Is v a function of x? We see at once that the criterion for v to be a function of x is that the particle should not reverse. Thus for motion with constant acceleration, v is a function of x if the initial velocity is of the same sign as the acceleration. For a particle executing simple harmonic motion, v is not a function of x. If, however, the particle describes only a half-cycle, so that, say,

$$x(\tau) = \alpha \cos \beta\tau; \text{ and the domain of } x \text{ is } [0;\pi]$$

then v is a function of x. We can, in fact, exhibit the function. If α is positive,

$$v = -(\beta^2(\alpha^2 - x^2))^{\frac{1}{2}}.$$

That is to say,

$$v = f(x)$$

where f is the function defined by

$$f(\xi) = -(\beta^2(\alpha^2 - \xi^2))^{\frac{1}{2}} \text{ whenever } \xi \in [-\alpha;\alpha].$$

Factoring Made Systematic

Senior secondary teachers will find this article useful if they have not already found a preferred method for teaching factoring. Our contributor is D.J. Davidson of L.V. Rogers Senior Secondary School, Nelson.

I believe the following to be a more systematic and meaningful approach to the factoring of trinomials than the one usually given in textbooks.

The product of two binomials and the application of the Distributive Principle should be well understood first. In general:

$$\begin{aligned} & (a + b)(c + d) \\ = \; & a(c + d) + b(c + d) \\ = \; & ac + ad + bc + bd \end{aligned}$$

One property of this result that we notice and will utilize is that the product of the two 'outside' terms equals the product of the two 'inside' terms. That is, $ac \times bd = ad \times bc$. In particular:

$$\begin{aligned} & (2x + 3y)(4x + 5y) \\ = \; & 2x(4x + 5y) + 3y(4x + 5y) \\ = \; & 8x^2 + 10xy + 12xy + 15y^2 \\ = \; & 8x^2 + 22xy + 15y^2 \end{aligned}$$

(Notice in the third line that $8x^2 \cdot 15y^2 = 10xy \cdot 12xy = 120x^2y^2$)

Now, to factor the trinomial $8x^2 + 22xy + 15y^2$ we just reverse the steps in the above example. The first problem (and the only one) is to find the two terms that were added to give $22xy$. But we use the fact that these two terms also have a product equal to $120x^2y^2$. So, (just using the coefficients now) we want two numbers that have a sum of 22 and a product of 120.

We try:

1 + 21	1 x 21 = 21
2 + 20	2 x 20 = 40
3 + 19	3 x 19 = 57
etc.	etc.
10 + 12	10 x 12 = 120

Some intelligent guessing would eliminate some of the trial guesses. Now we rewrite: $8x^2 + 22xy + 15y^2$
as $= 8x^2 + 10xy + 12xy + 15y^2$
and use the Distributive Principle to factor as follows:

$= 2x(4x + 5y) + 3y(4x + 5y)$
$= (2x + 3y)(4x + 5y)$

Another example: factor $4x^2 + 27x + 18$

(4 x 18 = 72, so we look for two numbers that have a sum of 27 and a product of 72.)

We try:

1 + 26	1 x 26 = 26
2 + 25	2 x 25 = 50
3 + 24	3 x 24 = 72

Hence

$$
\begin{aligned}
& 4x^2 + 27x + 18 \\
= {} & 4x^2 + 3x + 24x + 18 \\
= {} & x(4x + 3) + 6(4x + 3) \\
= {} & (x + 6)(4x + 3)
\end{aligned}
$$

And how about negative numbers?

Factor $6x^2 - 7x - 10$

(6 x -10 = -60, so we look for two numbers that have a sum of -7 and a product of -60)

We try:

$1 + -8 \qquad 1 \times -8 = -8$
$2 + -9 \qquad 2 \times -9 = -18$
etc.
$5 + -12 \qquad 5 \times -12 = -60$

Hence

$$\begin{aligned} & 6x^2 - 7x - 10 \\ = {} & 6x^2 + 5x - 12x - 10 \\ = {} & x(6x + 5) - 2(6x + 5) \\ = {} & (x - 2)(6x + 5) \end{aligned}$$

And a final thought on factors. Exercises and tests are mostly made up of a set of expressions, all of which can be factored. This is somewhat artificial and students should develop the ability to decide which expressions can be factored and which cannot. A better set of exercises would begin with the instruction, 'Factor where possible,' and would contain several expressions which do not factor.

Formalism and Creativity

The following was provoked by attendance at Surrey math teachers' professional day. Peter Bullen, UBC math professor, Bill Sims, BCIT 'subject x' teacher, and Sheilah Thompson, Douglas College counsellor comprised the panel. They and the audience of math teachers participated in a lively exchange. Mike Baker served as chairman.

Creativity? No, I can't tell you what it is, though I have many confused ideas about it. Nor can I say much about 'joy and beauty in mathematics,' except that there are such things. If you look closely at the work of our great ones, you certainly become aware that they possessed these things. And if you look at most of our children, you can be aware that they are capable of it. Then a teacher's problem is: How can I transfer my appreciation of these things, which is private to me, to my student in such a way that he is not limited by my appreciation?

And that, my friend, is a question! And I don't know the answer, any more than you do. I would like to suggest that there are some things we should not do which we do regularly.

I am reminded of the story of the good man who came down with pneumonia. The local witch-doctor said nothing, but bound a dead herring to the sole of each of the patient's feet. Within a few days, the man was hale and hearty. How did the medicine work? Several years of patience and much fire-water enabled his cronies to wring the doctor's secret from him. 'Whether the man lives or dies depends not on me but on his strength and will. As soon as he feels a little better, he will want to get out of bed, the worst thing he could do. Visitors will tax his strength too. You won't get out of bed if you have to walk on dead fish. Visitors will stay away.'

Witch-doctors, wizards and astrologers never divulged the 'how come' part of their work. 'If you will do what I tell you, the result will be a happy one,' is the best a layman could get from the wise ones. The mathematician, it seems to me, is often the modern counterpart of the wizard: 'Here is a set of definitions, this is what you can do with them, here is a proof which logically guarantees the consistency of my statements.'

For the teacher of young people, such an introduction to a topic is useful if he intends that his students shall learn to do always what they are told to do, and in the form he demands. But creativity cannot be engendered this way, for at the point of writing his definitions, the mathematician's creative work is finished. The definitions will set out the results of the creativity, but the thing itself is not on display.

Cayley does not tell us the kind of thinking process which enabled him to invent matrices. He simply defines the matrix and the operations with matrices, from which flows a magnificent sum of facilities in complicated operations. But the creative thought which produced the definitions is nowhere to be found; the definitions conceal *that* as well as any ancient wizard's secrets. In an earlier article I suggested what *could have been* a manner of thinking which resulted in the invention, because I am constantly bugged by students asking 'How come?' I feel this question represents a striving to be creative.

Nor does Newton reveal the kind of thing which was his creativity. We learn the derivative through a maze of definitions and theorems, the purpose of which seems to be to deny to the student the simplicity and beauty of the derivative itself. The original work is formally presented in Latin; it is thereby automatically unavailable to most.

If you need a more biting demonstration of how creativity can be concealed by the 'definition - use' presentation of a topic, look up John Nepair (nowadays spelled Napier), who in 1614 presented his invention of logarithms to the scholars of the time, in Latin. He was persuaded to permit and approve the English translation by Edward Wright which was published in 1616 under the sponsorship of 'The Right Honovrable and Right Worshipfvll Company of Merchants of London Trading to the East-Indies.' The merchants knew a good thing when they saw it: Napier's logs permitted the calculation of a position or course within an accuracy that was staggering in those days, within a ship's length on the surface of the globe, accuracy beyond the reach of the best instruments of the time; calculations which formerly occupied hours would be reduced to minutes, and would be less 'subject to many slippery errors.' There is little doubt that Napier played a large part in Britain's ruling of the waves in the centuries which followed. This book should be in every school library. It is published in facsimile by Da Capo Press, New York (1969).

But what is this work? A table of sines (numbers) with their logs for the angles of the quadrant to the nearest minute, with an interpolation chart invented by Wright, these sines and logs accurate to six significant figures. And a set of instructions on how to use them in common arithmetic operations and in plane and spherical trigonometry. I do not mean to detract from the magnificence of Napier's work when I point out that this is another example of the 'definition - use' introduction of a topic.

A creative youngster simply *must* ask: 'How did he get these logarithms?' All the good laird offered was to this effect: 'I have two number-lines on each of which a point moves from the end. On the first, in a given time interval, the point moves the same distance as it did in the previous time interval (the line of logarithms). On the other, the point moves a fraction of the distance it moved in the previous time interval (the number line). The points start at the same speed from the end of the line, and intervals on one line are to correspond with intervals on the other line.' From this picture he states his definition. But that all-important fraction, how did he choose that? He doesn't say. In language reminiscent of The Canterbury Tales (but without the naughty stories), he tells us he isn't going to say how:

> 'Hitherto we haue shewed the making and symptomes of *Logarithmes*; Now by what kinde of account or method of calculating they may be had, it should here bee shewed. But because we do here set down the whole Tables, and all his *Logarithmes* with their Sines to euery minute of the quadrant: therefore passing ouer the doctrine of making *Logarithmes*, til a fitter time, we make haste to the vse of them: that the vse and profit of the thing being first conceiued, the rest may please the more, being set forth hereafter, or else displease the lesse, being buried in silence. For I expect the iudgement and censure of learned men hereupon, before the rest rashly published, be exposed to the detraction of the enuious.'

The 'fitter time' never arrived. John Napier died in the year following the English publication, the substance of his creativity 'buried in silence,' the massive product of his creativity left as his monument.

Not only does the 'definition - use' introduction conceal from us his choice of that fraction mentioned earlier, he chose to make it difficult to find by making his logs decrease as the numbers increased.

We find his logs to be based upon 1/e, e = 2.71..., the trancendental number you and I learned about as a consequence of our ability to absorb some calculus. But calculus was invented a half-century *after* Napier's death.

There are three possibilities evident from this situation:

a) Napier chose a base related to e at random. This possibility is exceedingly remote; Briggs simplified the tables by choosing ten, as you and I would probably do;

b) His publication of logarithms was preceded by a set of investigations involving the ideas of calculus, but he was not ready to submit these to the 'censure of learned men,' so did not formalize them. I can find ln 2 or ln 3 using only arithmetic and the idea of 'a little change,' accuracy being limited only by patience. But I must use the ideas of calculus. Reading Napier convinces me these ideas were available to him, *taught by himself*.

c) If neither of these is acceptable, then there is a relationship between π and e which Napier discovered and which has not since been rediscovered, for angle measurement is essentially a circular operation. Had not the 'censure of learned men' been present he might have written about it so that others could continue his line of investigation.

One can forgive the witch-doctor and astrologer of old for guarding their secrets. Even in Napier's time, the censure of learned men was a severe thing, witness the recanting of Galileo and the burning of Giordano Bruno. If you wanted to publish, you had to set out form and substance acceptable to the great ones.

More than any other discipline, mathematics continues the formalistic 'definition - theorem - proof' presentation of topic material, before the ideas which give rise to it or the necessitites from which the ideas spring are discussed. Yet it is evident to any student of history that formalism and conformity have always been the enemies of creativity. Of recent years formalism has been used in the high schools to a greater degree than before in the belief that it fostered understanding, whatever that might be, for it is different in each of us.

Do you want your sick one well again? Don't let him waste his strength.

Do you want to teach creativity? Don't burden your student with formalism. Try to find examples of creative work to show him, best beginning with your own.

By the way, the name Nepair looks very much like nonpareil, which means 'there is none to be his peer.' I'll drink to that! -- Bruce Ewen.

A Pump Problem

Four Ajax pumps and three Bell pumps together pump as much water in five hours as three Ajax pumps and five Bell pumps together pump in four hours. Which type pumps at the greater rate?

The Five Trees

Five trees have been planted at equal distances round the circumference of a circle. It is the same distance from the willow to the pine as it is from the pine to the oak. The birch is nearer to the oak than the elm. Starting at the oak and traveling clockwise round the circle, you would reach the elm before the birch.

If you started at the pine and traveled clockwise, what is the first tree you would come to?

An Ancient Problem

King Croesus donated to the temple six golden bowls weighing a total of 1185 ounces. The heaviest bowl weighed one ounce more than the second heaviest which weighed one ounce more than the next heaviest and so on. What were the weights of the bowls?

Misfits

Find the 'misfit' in each line.

(a) 72, 36, 45, 63, 15, 27.

(b) radius, diameter, arc, chord.

(c) inch, metre, gram, litre.

(d) pint, quart, gallon, litre.

(e) vertex, allied, acute, reflex.

(f) 90°, -90°, 450°, -270°, 810°.