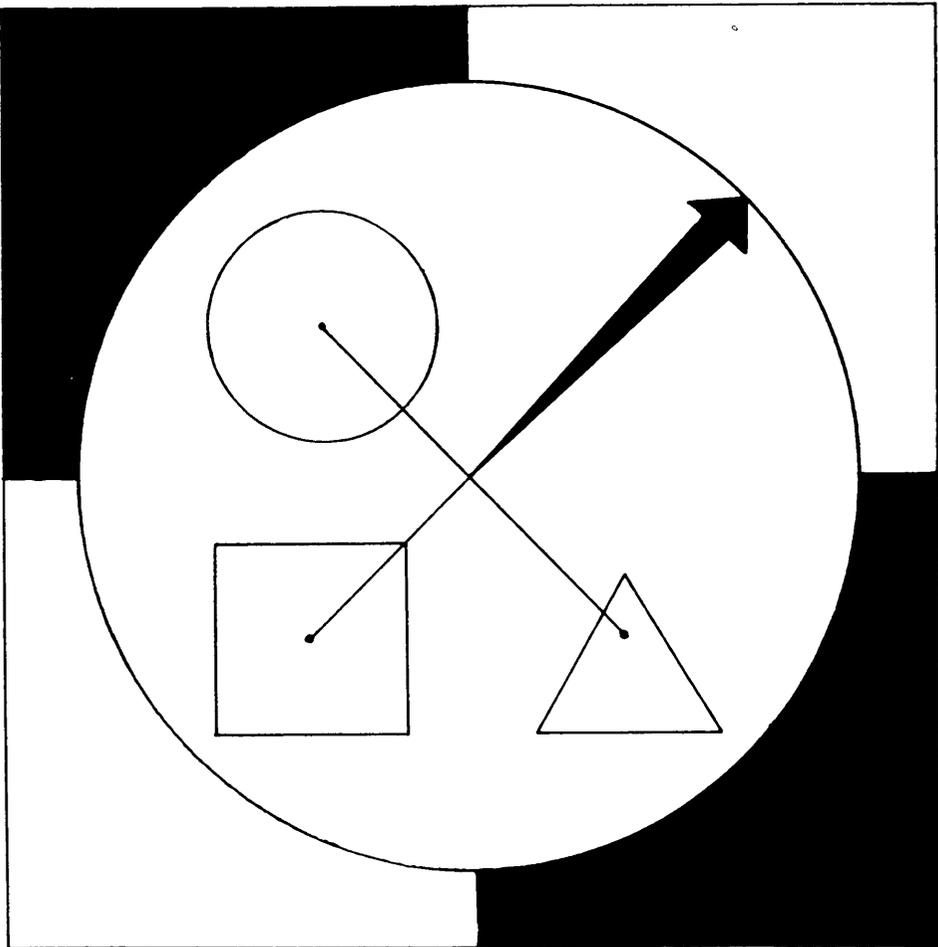


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Vector

BRITISH COLUMBIA ASSOCIATION OF MATHEMATICS TEACHERS

NEWSLETTER



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BCAMT EXECUTIVE 1971-1972

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The B. C. Association of Mathematics Teachers publishes Vector (newsletter) and Teaching Mathematics (journal). Membership in the association is \$4.00 a year. Any person interested in mathematics education in British Columbia is eligible for membership in the BCAMT. Journals may be purchased at a single copy rate of \$1.50. Please direct enquiries to the Publications Chairman.

MESSAGE FROM THE PRESIDENT

As incoming President, I should like to welcome new members to the B. C. Association of Mathematics Teachers, and to thank those members who rejoined the association. Since we are a very keen, but new, executive this year, we ask your tolerance for our errors and oversights. We hope to build an organization that will both represent and serve you.

My greatest concern is for any separation that exists between the executive and the membership. The BCAMT should *not* be regarded as a 'head office' organization. Its executive is made up of practising teachers and those directly concerned with mathematics education. If our organization is to be vital and relevant, the executive members should be personally acquainted with most of the membership. Right now, I can put faces to the names of fewer than 40 of our 450 members. This situation will change over the next two years, I hope.

This year's executive has given primary importance to an in-service program. The workshops on various themes and discussions of curriculum should bring the members of the organization closer together.

Although they are not formally organized into chapters of the BCAMT, there are a number of local math teachers' associations, department heads' groups and active, involved school groups that could and should participate on a provincial level.

If you as a member have something you wish to say to or to learn from the executive, please drop us a note. Also, you are most welcome to attend executive meetings. Representatives from larger groups might be considered in cost-sharing arrangements. The next executive meetings are October 23, 1-5 p.m., Education Building, UBC, and December 4, 9 a.m. - 12 noon, BCTF Building.

I hope that the BCAMT will become the kind of clearing house for information and ideas that you will appreciate.

ON CURRICULUM REVISION

At the September 18 meeting of the BCAMT executive, a discussion of the status of the on-going curriculum revision took place. As a result, it was decided that information should be requested from the Department of Education. A letter was written strongly encouraging the establishment of pilot projects, involving our members, before implementation of new programs. It was recommended that in-service opportunities be available to teachers prior to and following the introduction of any new programs. The in-service activities should be the joint responsibility of this association and of the Department.

We hope to be able to report back to you on the above in our next newsletter.

PLANNING UNDER WAY FOR 12th NORTHWEST CONFERENCE

Ralph McTaggart has undertaken the massive task of organizing the Twelfth Annual Northwest Mathematics Conference, to be sponsored jointly by the BCAMT and NCTM. Vancouver is to be the host city. McTaggart is mathematics department head at Kitsilano Secondary School in Vancouver.

If you can offer assistance, McTaggart will be delighted to hear from you. Write him at 6429 Marine Drive, New Westminster, or telephone 526-8230.

AN IN-SERVICE PROGRAM FOR 1971-1972

The BCAMT is now accepting requests for in-service workshops, to be held in locations convenient to any group of teachers. Teachers interested in convening such workshops should get in touch with Sandy Dawson, In-service Chairman, using the following address: Dr. Sandy Dawson, Professional Development Centre, Simon Fraser University, Burnaby 2, B. C.

Below is a list of suggested topics for such meetings, but it is emphasized that if your group has a specific topic it wishes to cover, arrangements can be made to meet your needs. Suggested topics for single-theme workshops are:

1. Teaching of fractions in the intermediate grades (or other topics in the elementary school);
2. Role and function of geometry in the junior and senior secondary school;
3. Creative teaching of algebra – elementary and secondary school;
4. Base-x Arithmetic – a modern heuristical approach to the teaching of algebra;

5. What mathematics for the non-college-bound student;
6. Articulation of elementary-secondary programs;
7. Use of the activity approach to mathematics;
8. Individualization – use and abuse of.

In addition to these single-theme workshops, the BCAMT plans as soon as possible to sponsor a 'mini-conference,' to which will be invited individuals responsible for mathematics programs in the elementary, secondary and post-secondary institutions. If plans mature, the conference will have as its purpose to establish and foster communication among mathematics educators from various levels of schooling and a variety of post-secondary institutions. If you are interested in contributing to a conference of this kind or have ideas for the 'thrust' of such a gathering, make your interest know to Dr. Dawson.

A revision of the mathematics curriculum K-12 is imminent. The changes to be recommended, the program to be followed, have implications for *all* teachers of mathematics whether at the elementary, secondary or post-secondary levels. No more need be said to emphasize the need for a good in-service program in the months ahead.

SUPPLEMENTARY PUBLICATIONS

Members are reminded that the BCAMT is trying to sponsor the preparation and distribution of brief monographs that will be useful to teachers of mathematics. The first of these, a programmed approach to Binomial Expansions, is available as BCTF Lesson Aid No. 3028. Teachers of Math 12 should consider ordering a class set.

Teachers throughout the province are invited to send manuscripts on materials they find useful in their classes, particularly those that help to explain concepts and could be worked into a short monograph on the topic. The committee does not wish worksheets, etc., except in cases where they are an integral part of the discussion.

At this time we should like to ask any teacher who would be willing to help in the work of editing these manuscripts to contact the chairman, Geoff Horner, 33675 Marshall Road, Abbotsford. He will then be asked to examine a manuscript, try out the idea in his classroom, and make suggestions that would help the author to improve it.

Your help is desperately needed, so please send your name, and the courses for which you would be willing to read a manuscript.

— Geoff Horner

STUDENT-TEACHER CORNER

The BCAMT extends an invitation to its student-teacher members to become involved in its publications. (No, it is not true that all teachers regard student-teachers as idealistic, inexperienced and incompetent!)

Because of the time and thought put into them, the projects, reports and lesson ideas that you develop as a student-teacher are worth printing. So what if you don't have an idea polished to perfection? Any practising teacher worth his pay will adapt a good idea to suit his own situation. Send us a copy of your work.

This may be the only opportunity for students from UBC, UVic, SFU and Notre Dame to exchange ideas.

A NOTE TO UBC STUDENTS IN ED. 440 (COMPUTERS):

Last year one of the Ed. 440 students, Keith Robertson, undertook as his project to work with some Grade 8 and 9 students at Wm. Beagle Junior Secondary School in Surrey. Two of the Grade 9 boys took Computer Science 210 during summer session. Both got first class marks; and one tied for top mark (97%) on his final examination. Their thanks go to Keith for the time and effort he gave to preparing them. Keep up the good work, 440 students!

A PUBLICATION FOR STUDENTS

Issue No. 2 of *Student Mathematics*, written by and for students of math, has just been received. Those wishing to go beyond regular curriculum work, or those interested in a math club, should have this publication available to them.

The new edition contains, among other articles, an account of a study of perfect numbers in binary notation. Its author is Grade 11 student Lyle Craver of Carson Graham Secondary School, North Vancouver. The editor, Dr. W. W. Sawyer, requests your students' contributions.

Requests for copies should be addressed to The Secretary, *Student Mathematics*, Room 373, College of Education, 371 Bloor Street West, Toronto 181, Ontario. Single copies are 10 cents if you enclose with your request a 4 x 9 stamped, self-addressed envelope. Quantity orders are 10 cents a copy, plus 50 cents mailing and handling charges.

BEATING THE BLACK BEAST

Tom Bates recently referred to the courses of 'general math' as the bête noir of teachers. John Cuthbertson of West Vancouver Secondary School found a way of producing a course of value to his students. Let him tell it.

I came across my model while attending a teachers' convention in the U.S.A. The textbook, *Advanced General Mathematics* by Eldert Groenendyk (Central University of Iowa Press, Pella, Iowa), consists of 30 projects. The author wrote various businesses or other employers in the region, asking them to outline typical problems an employee would be expected to solve and to describe the work done by the company. In exchange, he incorporated the company's letterhead into the lesson as finally prepared. Each project is thus a study of the operation of a particular company or agency, complete with the math it finds necessary.

I found this procedure worked for me:

1. For the first part of the term (until the end of October) I worked with important topics selected from the text, teaching in the normal fashion to establish control.
2. I arranged with the teacher of business machines to give two lessons (or have competent students give them) on the use and misuse of machines.
3. I introduced projects as individual work, issuing work sheets outlining assignments and indicating directions of research.
4. Students were then free to work at their own rate and to hand in projects as they were completed. Comment sheets were made out (by me) in duplicate; one of these was filed. Projects satisfactorily completed were discussed with the student and then filed (to prevent cribbing). Projects poorly done were returned to the students for completion. A student's term mark depended on the number of projects completed.

Students were free to research in the library or (more usefully) at the location of the company doing the business outlined in the project (clearance was obtained through the school office for travel by student's car). Most students worked individually, but some worked in small groups. The latter tended to lack initiative and to do less well.

I have found this approach productive. I hope others may find it of some help.

Mr. Cuthbertson is spending this year as an exchange teacher in England.

CARDS, ANYONE?

The use of games instead of drill increases in popularity. Games can be invented to correspond to the concept of your concern. The following is adapted from an article by Stephen Krulik in the New York State Mathematics Teachers' Journal, June 1971.

Card games are essentially mathematical in nature, but games of bridge or poker may not yet be acceptable math class activities, especially if there is money on the table. Simple games, such as fish, casino or rummy, are easily adaptable to the lesson you are teaching, the 'deck' easily made.

Fraction War

To practise comparing values of fractions. Sixty-six cards about $2\frac{1}{2}$ " by 3" (half a 3" x 5" index card will do) are marked, using a felt-tipped marker, with a fraction on each as follows: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{1}{5}$, . . . Do not reduce to lowest terms. Shuffle and deal entire deck face down to any number of players up to six. To play, each player turns up top card. High card wins and winner places all exposed cards face down at the bottom of his deck. To break a tie, tied players turn up another card each. Winner is the player who has the most cards at end of game. Game ends if a player runs out of cards.

Factor Casino

Casino is essentially a matching game. Prepare a 45-card deck by marking in black on each of 20 cards an algebraic expression to be factored; mark in red on another card the factored form of each of these. On each of the remaining five cards mark in red some incorrect factored form, preferably common student errors.

Shuffle and deal five cards to each player and five cards face up on the table; the remainder of the deck is placed face down on the table. Play proceeds around the table in the usual fashion. A player takes a trick when he matches an expression from his hand with its alternate form from the table. Tricks are held in front of each player on the table, face down, in a manner that permits them to be counted easily. If a player takes a trick, he replaces the card from his hand with the top card from the deck. If he cannot take a trick, he must place one card from his hand face up on the table and replace it with the top card from the deck. If a card placed on the table matches one already there and the player discarding it does not notice the matching, the trick belongs to

the first player who notices it. A renege occurs if an incorrect matching is made, and the trick belongs to the player who calls the renege. Game ends when deck is finished and there are no more tricks to be taken. Winner is the player who has the most tricks.

Factorummy

A little more complex in play, this game follows the usual rummy pattern. Fifty-two cards are prepared in four suits, using differently colored felt-tipped markers. Each of five cards in a suit will be marked with polynomial expressions to be factored (for best result some should require three factors, even if one factor be a constant). The remaining eight cards in each suit are marked with a factor each. All suits are marked the same, but each in a different color. Cards are dealt seven to a player, face down; the remainder of the deck is placed on the table, face down, with the top card turned up, as in regular rummy.

Each player attempts to arrange his hand so that it consists of groups (three or four cards having the same expression) or runs (a polynomial expression together with its factors).

Play proceeds thus: each player in turn chooses either the top card of the face-down deck or the top card of the face-up deck on the table. He adds this card to his hand. He then discards from his hand one of the eight cards he now holds so that between plays his hand consists of seven cards only. Play ends when a player has no card that does not belong to a run or a group, but no card in that hand may belong to a run and a group at the same time. The winner, of course, must declare the win by the use of the word 'rummy.'

Each of these games will require a deck for each five players or thereabouts. The level of difficulty can be tailored to suit your needs, all the way from practice in addition facts to identities in trigonometry. Cost is practically nothing.

If your students enjoy this form of activity, they will invent their own variations. Listen well; you are about to meet the *real* mathematicians.

A Book Review:

MATHEMATICAL SENTENCES

MATHEMATICAL SENTENCES, Unit 3, of the NCTM series *Experiences in Mathematical Discovery* (1966) is a 35-page booklet supposedly for General Mathematics, but is a very useful extension of Grade 8, Chapters 17 and 19, as well as a good preliminary for Chapter 20, 'Story Problems.'

The original intention of Chapter 20 was to develop skill in writing down but *not* solving equations. Without going into the weaknesses of the present Math 8 text with respect to the content of Math 7 and the pressures of Maths 9 and 11, etc., some teachers believe that Math 8 should include an introduction to equation-solving.

This booklet leads into equations of such difficulty as $17x - 11 = 40$ or $(2m - 4)/5 = 6$. The approach is gradual and the exercises sufficient that even most below average students have some success. The intuitive idea of 'doing and undoing' (inverse operations) is stressed rather than the axiomatic treatment used in Math 9. Students gain experience in writing equations from word problems and solving these equations.

The book is well worth the 50-cent price. One could obtain a class set for less than \$20. (A rarity these days!) Books can be obtained locally from T. A. Howitz, Faculty of Education, UBC, Vancouver 8.

— Mike Baker

PROVE IT! WHAT DO YOU MEAN, SIR?

In which the writer suggests that we take a good look at the things WE do. Your comments will be welcomed.

1. Prove each of the following:

A. $\forall x, 3x + 15x = 18x$

- *Modern Elementary Algebra* (Nichols, Collins, MacPherson), page 124.

The above question is just one example of a type that abounds in our textbooks. I am not sure teachers are in complete agreement about the main intention of using such questions in the mathematical education of persons 13 to 15 years

of age. I propose that we ask ourselves why we should spend time on such material.

We could be using this question, or any like it, for one of the following reasons:

- (1) It is our intention simply that the student establish the truth or falsehood of such a statement.
- (2) We intend to provide a set of circumstances from which the youngster may discover some fact about the number system we use.
- (3) It is our desire to kindle in the youngster an appreciation for the method of thought commonly referred to as logical, and provide him with an opportunity to succeed in the exercise of postulational thinking.
- (4) We hope to achieve more than one of these objectives at the same time.

I know of no other valid reason for a class to use time discussing such question as the one above. Let's examine these possible objectives in terms of the possible gain to a youngster.

Proof can be too easy

Intention 1 above is a valid teacher objective only if it is preliminary to some other intention. It is nice to know that the thing you are trying to prove is true *before* you try your hand with logic. Even if it had value as an objective in itself, the demonstration of truth or falsehood is so often obvious that the youngster cannot take the 'proof' seriously. Here's one proof you probably haven't used:

If $x = 1$, the statement says $3 + 15 = 18$ and is true;

If $x = 3$, the statement says $9 + 45 = 54$ and is true;

The statement $3x + 15x = 18x$ is therefore true for all x .

How come? The statement is either true for no x 's (a nullity, the set is empty), for a finite number of x 's (an equation), or for all x (an identity). I have shown it true for two values of x , one more than is permitted for this statement to be an equation by the fundamental theorem of algebra. I know of teachers of algebra at the junior secondary level who never heard of this theorem. It's not in the curriculum at the level they teach.

I wish to emphasize that Intention 1 by itself has little value as a teacher objective. It cannot be the reason such exercises are included in a course in mathematics.

What about Intention 2? I think we can agree that this is always a valid objective for a teacher. I disagree with the idea that we can find out facts about the number system by the logical method. To find facts we must go into the number system itself by experimentation. When we do this, we find those facts that are expressed in the axioms and postulates we use. Memorization of postulates is no substitute for the finding of empirical facts.

Can it be proved at all?

It is not possible to 'prove' that $1 + 1 = 2$ unless you accept the Russell definitions of the number 'one' and the number 'two.' These definitions were devised so as to present the number system as a logical structure, it being known that the number system we use is not logical in any formal sense, having been in very useful existence millenia before the invention of formal logic. In the same way the postulates of Birkhoff incorporate the art of measurement into the logical structure of Euclid's geometry. The Russell definitions are as circular as they can be. The fact that they have been hailed as profound by so many mathematicians is probably one of the reasons the twinkle stayed in the grand old man's eyes so long. By using them, says Russell in his *History of Western Philosophy*, you can 'prove' that $2 + 2 = 4$, 'but the proof is long and tedious.' Without them you can prove nothing about the numbers of arithmetic, the foundation of our number system.

Now, if you cannot prove $2 + 2 = 4$, what about $30_{\text{ten}} + 150_{\text{ten}} = 180_{\text{ten}}$, which is much more difficult? Or what about $30_{\text{twenty}} + 150_{\text{twenty}} = 180_{\text{twenty}}$? How, then, can you hope to prove $30_x + 150_x = 180_x$, the same problem in any base, which is another way of writing the statement $3x + 15x = 18x$?

Hence, if you say you have proved the statement, you are saying you have tested this idea against other ideas accepted previously. *You have tested your own ideas*, but have demonstrated nothing about the number system you did not know before.

If #2 is your objective, you will find experimentation the best method, one that can be understood and enjoyed by most students, provided you have done your homework and have arranged good experiments. For an experiment to be a good one, it must permit the student to find for himself the facts you want him to find. But you are kidding yourself if you expect students to discover a fact of the number system he did not know using the method of logic. And do not expect his expression of a discovered fact to be in the language of the textbook.

It is often said that the math teacher shouldn't tell the student everything; he

should find out for himself. To apply this important maxim of good teaching correctly, the teacher must provide situations from which a student will be able to discover something. Unless the teacher does this, application of the maxim will become the 'mushroom farm' method of teaching. Keep that in mind if you say your intention is covered by # 2 above.

Focus on student, not subject

Reason 3 is the ultimate objective of any mathematics teaching. Few students will require in life all the number facts they can discover in a secondary school math course, but an orderly method of thinking is essential to every member of any society. A person's freedom, his ability to participate in society, depend directly upon his ability to think effectively. No one will question the validity of Reason #3. But if this is the objective, it must be clear in the teacher's mind that the student is the center of the exercise, not the facts discussed in the argument. The student will be most successful if he participates knowing that *he* is the object of the exercise. In most cases, this is not so. Think back on your own experience – were you aware when you proved a theorem in geometry that your ability to think was the central issue, not the pair of triangles? Didn't you already know the theorem was true before you 'proved' it?

Objective #3 will be defeated if the student does not already have the number-facts involved in the proof at his disposal, and with a degree of *fluency*. It will be defeated, also, by insistence upon the use of symbolism in any way other than as a means of shortening the statements. Just because the mathematician delights in using symbolism is no reason for causing the neophyte to be defeated by being required to use it before it suits his purposes.

If you subscribe to objective #4, my advice is, forget it. Very young students can and do discover the facts for themselves; to produce the logical argument concerning those facts is a mark of relative maturity; to produce the argument with all the hairsplitting, exclusions and inclusions found in textbooks is the kind of thing one might expect from a mature adult, not a teenager. If you wish to teach the fact and the logic with which it can be surrounded at the same time, either the fact has not been taught soon enough or the logic is being taught too soon. In either case a gross error has been made, and the youngster's ability to proceed further, meaningfully, is impaired. Pythagoras could not have written his theorem had not the fact been employed by builders for hundreds of years. The individual may not generalize until after he has found out the facts. When he can generalize in the smallest degree, more facts are more accessible to him.

I think it not important that we should agree on why we take up such questions. I think it exceedingly important that each teacher make up his own mind which of the four objectives is his objective. The answer can be different for different individuals in the same class. Once he decides what his intentions are, he can make the available resource material (usually a textbook) serve those intentions. The responsibility in this matter belongs entirely to the teacher, and is not lessened if he considers the text unsuitable.

The alternative is to wait until a curriculum and its set of prescribed texts entirely suit every one of us. If that ever happens, the textbook will do the teaching job.

-- Bruce Ewen
