Vector

BRITISH COLUMBIA ASSOCIATION OF MATHEMATICS TEACHERS

NEWSLETTER

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This space is still reserved for a design created by one of our members. A few designs have come in, but we are still looking for more.

SEND IN YOUR DESIGN SOON.

B.C. ASSOCIATION OF MATHEMATICS TEACHERS

Executive Committee

The B. C. Association of Mathematics Teachers publishes *Vector* (newsletter) and *Teaching Mathematics* (journal). Membership in the association is $3.00 a year. Subscription rate for all publications in one year is $4.50 for those persons not eligible for BCAMT membership. Journals may be purchased at a single copy rate of $1.50. Please direct enquiries to the Publications Chairman.

EDITOR

FROM THE EDITOR

We have a name, tentatively. Just to try it out for a while, we have given the newsletter the name VECTOR. If you like it, let us know. If you don't, find a better one and tell us that.

The design for the front page is still undecided. One design was submitted, but I should like to be able to choose from among a number. So, if you have a few minutes with your class, why not give them a chance to dream up something? You may be surprised by their artistic abilities.

There are some interesting items in this issue:

A Math Lab for Underachieving Pupils
Mathematics Typewriter Symbols
Computer Topics
First Year Courses at UBC

You will also find two enclosures with this issue:

1. NCTM Recent Publications
2. Mu Alpha Theta Mathematical Booklist

   Your executive thought that all mathematics teachers should have a copy of this list and therefore provided funds to purchase 600. PLEASE BE SURE YOUR LIBRARIAN HAS A LOOK AT IT.

REPORTS

1. Executive Meeting of October 26.

   At this meeting your executive decided:
   - not to sponsor the Ontario Junior Math Contest
   - to enter into an agreement to host the North west

Mathematics Conference every third year
- to obtain copies of the Mu Alpha Theta Booklist for all members
- to write to the Curriculum Directors requesting them to contact the Department again concerning the establishment of a Revision Committee.

2. Executive Meeting of December 7.

Publications: The Publications Chairman reported that he hoped to publish four more newsletters and two more journals.

Annual General Meeting: Plans are to hold the meeting at the Hotel Vancouver on the afternoon of April 7. Len Gamble is planning a full program for that afternoon.

3. Math Contest:

Art Olson of New Westminster Secondary School agreed to take on the difficult job of organizing the Math Contest. We shall undoubtedly hear more from him later on.

4. Journal:

The next issue of the journal should be out at the end of January.

5. Constitution:

If you wish a copy, write the president or the secretary. If you have any suggestions for revisions, let us know before the next executive meeting. (February 1)

(Some sweeping changes are being made in the mathematics program at UBC. The First Year changes have been passed by the Faculty of Science and will go into effect in the 1969-70 session. Dr. James sent us the following description of the new First Year courses.)

Proposed Program of First Year Courses at UBC

The proposed new Math 100 (2 units), 120 (1 unit) and 121 (1 unit) are the components of a basic introductory 4-unit course in mathematics. Although strongly related, they can be taken independently (the only exception being that Math 120 would make little sense without Math 100). Students who have already made up their minds about the program they wish to pursue may take only those components, if any, needed for that program. Students who are still undecided and wish to keep the opportunity of taking a heavy dose of advanced mathematics will be advised to register for the new Math 100 and Math 121 when entering the university and to wait until the end of the fall term before deciding whether to take the new Math 120 in the spring terms of their first year. Math 120 will be a prerequisite for most advanced analysis courses, and should be taken as early as possible by students who wish to pursue such courses during their undergraduate year.

Students who want only a general exposure to mathematics should consider taking Math 130 which is not prerequisite to further mathematics courses.

The proposed new courses for first year are as follows (Numbers in parentheses represent the number of units):

100 (2) Calculus I

(Meets 3 hours a week throughout the year.)
Prerequisite: Mathematics 12 (Secondary School Program, B. C.). Ideas, techniques and applications of differentiation and integration.

120 (1) Introduction to Analysis I

(Meets 2 hours a week for half a year. Offered in the fall and again in the spring term.)
Prerequisite: Math 100, which may be taken concurrently when Math 120 is taken in the spring term.
Discussion of some basic concepts underlying Calculus, such as induction, greatest lower bound, least upper bound, sequence, limit, continuity; and proofs of some theorems.

121 (1) Introduction to Vectors and Matrices
(Meets 2 hours a week for half a year. Offered in the fall and again in the spring term.)
Prerequisite: Mathematics 12 (Secondary School Program, B. C.). Systems of linear equations, vectors, matrices, determinants, linear dependence.

130 (3) Finite Combinatorial Mathematics
(Meets 3 hours a week throughout the year.)
Prerequisite: Mathematics 11 (Secondary School Program, B. C.). Intended primarily for students not in the faculty of science, who wish to have some exposure to mathematical thinking. At the discretion of the instructor, the course will discuss such topics as: permutations, combinations, the binomial theorem, probability, graphs, properties of numbers, geometric configurations, etc.

140 (1) Introduction to Linear Programming
(Meets 2 hours a week for half a year. Offered in the fall and again in the spring term.)
Prerequisite: Math 121
Linear programming problems; dual problems; the simplex algorithm, solution of the primal and dual problems; some special linear programming problems such as transportation, network flows, etc.

Further information may be obtained from the Department of Mathematics, University of British Columbia.

M. A. A. Contest

The examination for this contest is to be given on Tuesday, March 11, 1969. Registration fee for the contest is $4.00. (This fee includes the award pin for the highest ranking student in the school.) Each examination costs 10 cents.

Since the registration deadline is January 15, 1969, you should write immediately to:

TOM BROWN,
M. A. A. CONTEST CHAIRMAN,
DEPARTMENT OF MATHEMATICS,
Simon Fraser University,
BURNABY 2, B. C.

Continuous Progress

Two ideas making the rounds these days concern math labs and individualized instruction. We are reprinting an article on math labs. Individualized instruction is another kettle of fish, however. Some schools in the province have set up a program for continuous progress. If your school has done so, why not write an article for the newsletter? For most teachers in secondary schools, continuous progress is something that exists only in the elementary school - yet continuous progress will become a way of life in the secondary schools in the near future. Give it some thought.

MATHEMATICS TYPEWRITER

Besides being our capable president, Peter Minichiello has the time to write about his typewriter.

Does your clerical staff shudder and pretend not to see you when you approach them with a test or work sheet to type? Does the mimeographed product contain hand-drawn symbols which are unrecognizable by the pupils? If so, you (or your school) need a mathematics typewriter.

Obtaining a typewriter with a suitable keyboard is, however, no easy task. To my knowledge, no typewriter manufacturer produces a typewriter containing all of the modern symbols.

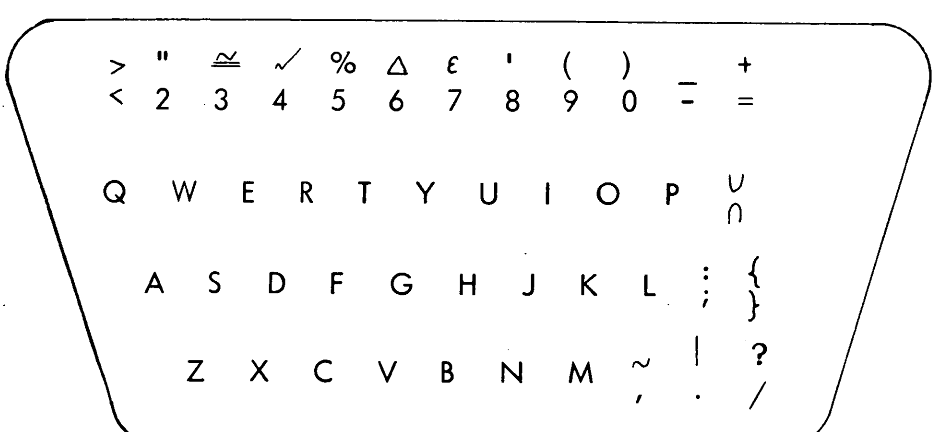
'Typit' symbols are available for use with any typewriter that can be fitted with a special attachment for holding the symbols. The custom-made attachment costs $9.00 (plus an installation charge) and each Typit symbol costs $4.95 (with no discount allowed to individuals or to schools). A set of Typit symbols sufficient for the needs of our courses would cost about $125 plus tax, attachment and installation charges.

Relying on Typit symbols for all mathematical typing is not as speedy or as convenient as the manufacturer would have one believe and, on some typewriters, the ribbon catches on the Typit and several strikes are necessary to type one symbol.

The keyboard illustrated is a modification of the standard Smith-Corona keyboard. The design was produced with the help of a patient and sympathetic typewriter salesman who was also a capable repair man. Some of the standard typefaces were split and the required mathematics symbols soldered in place. Most of the symbols were available from different standard keyboards made by the manufacturer, but $\cup$ and $\cap$ were cast to order. As I purchased the typewriter new, the modifications were made at no extra charge. Care was taken to ensure that a minimum number of standard symbols could be employed to produce a maximum number of 'made-up' symbols. For example, the $\surd$ was chosen to line up with / to form $\surd$ and $\sqrt{}$; the S and | combine to \$; > and _ to form $\geq$, etc. Only a few Typit symbols are needed in conjunction with the keyboard to produce almost all of the mathematical expressions used in our courses.

The keyboard has the distinct advantage of being suitable for any regular typing job as well as for its specialized function.

MATHEMATICS KEYBOARD



Specifications:
All-electric; 15" carriage; $\frac{1}{2}$-space; $\frac{1}{2}$-line;
Equipped with 'Typit' attachment for special symbols:
α, β, θ, π, $\subset$, etc.

Made-up symbols:

$\leq$ $\ngtr$ $\perp$ $\parallel$ $\neq$ $\therefore$ $\approx$ \$ ¢ $\angle$ $\div$

$[\;]$ $\sqrt{\;}$

Sample statements:

$f = \{(x, y): y = 3x^2 - 4x + 7, x \in R\}$

$\triangle ABC \cong \triangle XYZ$ $\overline{AB} \parallel \overline{XY}$ $\overline{MN} \perp \overline{RS}$ $m\angle A = m\angle B$

$\triangle ABC \sim \triangle XYZ$ $\{(x, y): \frac{x^2}{16} + \frac{y^2}{9} = 1\}$ $\sqrt[4]{\frac{3x}{8}}$ $\log_2 64$

$\cos(x - \frac{2\pi}{3})$ $\tan 2\beta = \frac{2 \tan \beta}{1 - \tan^2 \beta}$

A Mathematics Laboratory for Underachieving Pupils

by: NORMAN COWAN, Teacher Co-ordinator at the McAteer Mathematics Laboratory, at Belvedere Junior High School, Los Angeles.

In 1966 the California legislature approved the McAteer project, a bill allocating money to finance experimentation by individual school districts in new ways by which under-achieving pupils might learn the structure and skills of mathematics. The State approved three directions as proposed by the Los Angeles City Schools: closed circuit television, programmed learning, and a mathematics laboratory. In conformance with guidelines set by the state, the UCLA Center for the Structure of Evaluation of Industrial Programs, an outside agency, was asked to evaluate the program. This report reviews a few of the activities during the first year of operation of the Mathematics Laboratory.

One of the most important concepts of the mathematics laboratory included experimentation, recording of data from the experiments, and drawing conclusions from the data. The team of teachers believed that activities involving the sense, provided they were confined solely to teacher exposition and/or pencil and paper work from a text or chalkboard problems, fell properly into a mathematics laboratory classification. Each of four teachers taught two classes, using the remainder of the school day for developing new laboratory techniques. Supporting personnel included two full time clerk-typists, a part-time illustrator, and a part-time audio-visual technician.

Equipment ranged from simply checker type counters to a small but efficient desk top computer. Duplicating and projection devices were always available as well as audio-taping and playback.

The drawing of two vectors under a number line to show addition or subtraction is not a laboratory experience, while making a slide rule with linear scales to add or subtract, is preferred. Counting vertices on a grid of five parallel horizontal and four parallel vertical lines is not a laboratory method of illustrating 5 x 4 = 20, while cutting five slots on one cardboard and four on another, so that they may be placed together to show 5 x 4 = 20, is. In both illustrations, the laboratory device is manipulative and can be used to provide enough examples to generalize the concept.

Pupils often start school with knowledge which can surprisingly hinder the acquirement of further knowledge. As an example, consider the pupil who can readily count but who has not grasped the distinction between number and numeral. You may indeed ask that he match a set of pencil marks in one-to-one correspondence with some other set. The simple truth is that, due to the previously developed skill in counting, pupils naturally count, first the comparison set, then the matching set.

A laboratory experience can be devised to circumvent this previous skill. Let each pupil have sets of small, diverse items such as paper clips, caroms, and toothpicks. Prepare a tape using three distinct sounds such as a typewriter bell, a snare drum tap, and a sneeze. The bells, taps, and sneezes should follow one another in non discernable and rhythmic pattern. Pupils should determine the number of each sound heard on the tape by matching their sets of paper clips, caroms, and toothpicks in one-to-one correspondence.

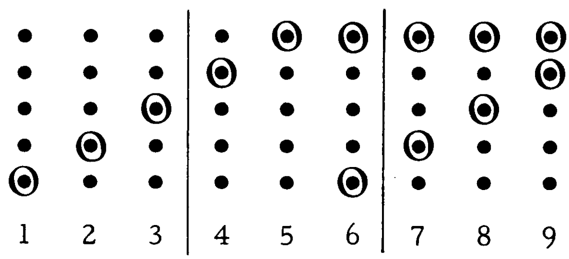
This multi-sensory approach could be used to solidify one-to-two, one-to-three or any ratio matchings. Almost every school has a pupil whose skill with the snare drum is sufficient to replace each bell sound. For example, with a group of three taps and each sneeze with a group of seven. If the room contains a control panel of several light switches, light flashes could be used in place of the sound tape. Three distinct movements such as hand claps, finger snaps and deep knee bends can vary the routine as well as provide some much needed movement.

Additional laboratory activities provided multi-sensory experiences involving place value. For example, simple models of rockets were constructed using different colored sticks and fasteners. Each model became a symbol for a number when a code was presented assigning the number of each color stick and fastener as the face value of a place value. The same model was used to symbolize many numbers by changing the code. Inverting the process, the pupils were given codes and asked to construct models to symbolize given numbers.

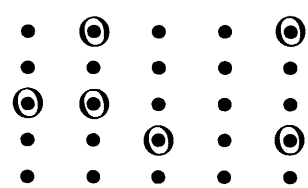
Typewriters were used by pupils to symbolize place value numbers using any typewriter key to make vertical rows for such numbers as 3, 4, 8 and 2.

```
    a
    a
    a
    a
  a a
a a a
a a a a
a a a a
```

Boards with checkers in vertical rows provide a similar and manipulative method of representing place value numerals. If all but the top checker is removed, a still more sophisticated manipulative number symbol can be made. The connection with the abacus is obvious and this device played its part in the unit. A geoboard and caroms proved to be the most challenging method of number symbolization. Each vertical column of nails represented place value. One or two caroms were placed on the nails to represent face value in the following way:



Pupils represented given numbers on the geoboard or read numbers represented such as:

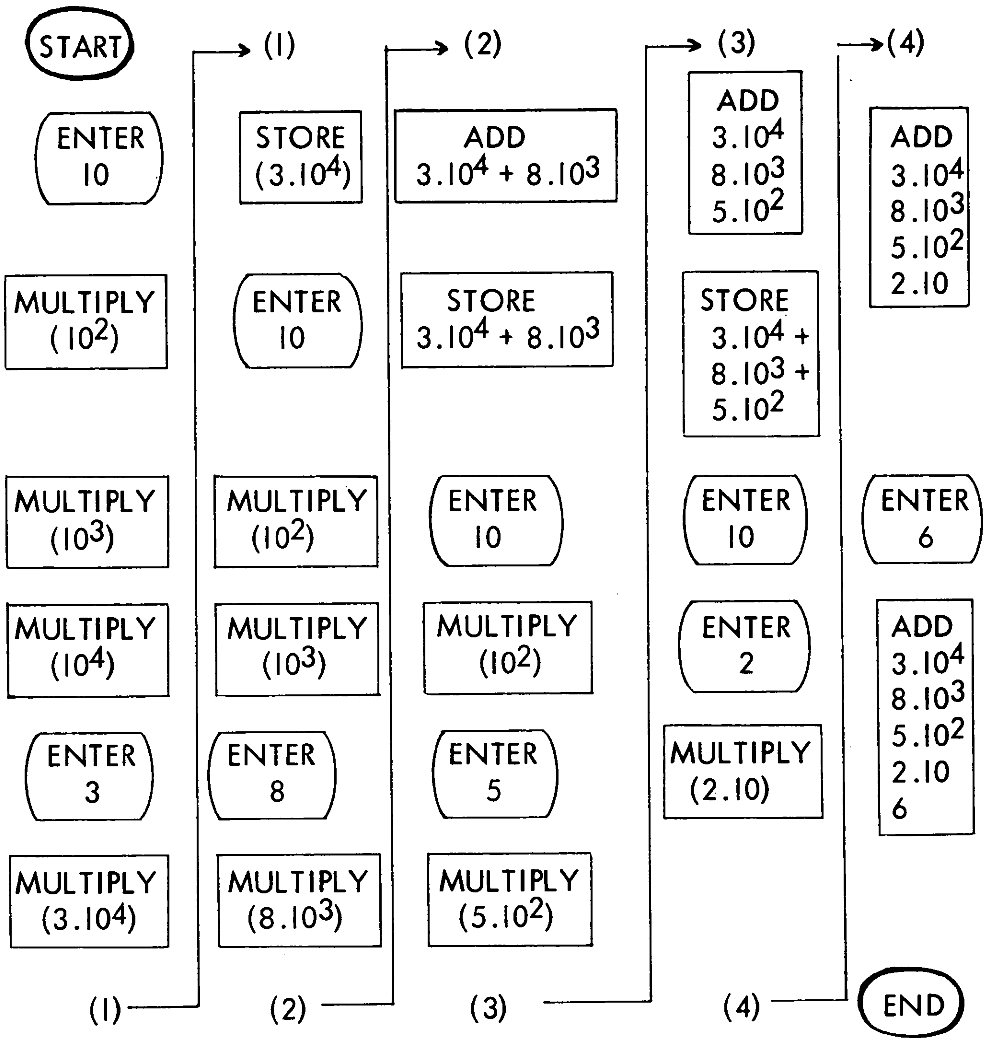


= 38, 207

Extensions of this procedure to graph paper are obvious.

Calculators and the computer were used for a place value game. The object of the game was to make the machines print large decimal numbers. Rules of the game confined the keys to be used to numberal keys no larger than 10 and the addition and multiplication key. Pupils flow-charted their strategy as follows:

OBJECT: Print 38,526



The calculators used had a Grand Total Register allowing storage of the sums. The computer could store the base 10 and face value digits.

The evaluation of the place value unit also became a game. Pupils were called secret agents. All instructions were given on a prepared tape by 'the chief.' The chief's performance and script were suitably (perhaps overly) melodramatic with references to enemy agents and counter spies.

Envelopes were distributed to all agents. The envelopes contained instructions directing pupils to different stations each of which contained a representation of a place value numeral of the type described above. Some of the stations used physical representations which were new to the pupil.

One such station required the use of the sense of feel. Sets of objects of different shapes and textures were placed in a box with an opening large enough for a hand. Each set of the same shape represented place value while the number of that set represented face value. Activities at the stations were interrupted from time to time by taped messages reporting the enemy's whereabouts and activities. This information was a code transmitted by reading large numbers. The pupil agents had to write these numbers in decimal notation on the appropriate forms found in their envelopes.

The evaluation ended by taped melodramatic message stating that the enemy had discovered the operation. We followed plan RED ALERT which was to place all papers back in the envelopes, hand one envelope back to agent 1 (the teacher), and, at the sound of a bell, walk quietly and unobstrusively to their next class just as if nothing had happened.

Pupils were involved and interested in these activities and the enthusiasm extended into mathematics acitivites in the laboratory, thus creating a climate for greater indepth learning.

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LETTERS

November 9, 1968

Dear Gerry

I'm glad you were able to use my piece on factoring, but would like to print out some inadvertencies which detract from its value:

P11 line 2, first factor should read $(a+3\frac{1}{2}b+\frac{1}{2}b)$
P11 line 14, factors should read (both) ' $\sqrt{7}b$' not '$\sqrt{7b}$'

I've already been challenged on the line 'These can be shown to be necessary and sufficient conditions.'

Here goes:

Given: In ax^2+bx+c, a is a perfect square, c is a perfect square, and $a+b+c$ is a perfect square.

Prove: ax^2+bx+c is a perfect square.

Proof: Since a is a perfect square, it can be written A^2. Since c is a perfect square, it can be written C^2. Then $a+b+c$ can be written A^2+b+C^2. Since $a+b+c$ is a perfect square, it can be written $A^2\pm2AC+C^2$.
But then $b=\pm2AC$ and hence $b^2=4A^2C^2$. That is to say, $b^2=4ac$ and the discriminant of ax^2+bx+c, $b^2-4ac=0$, the necessary condition for a perfect square.

I believe that if teachers would use only the ideas on the sums of coefficients of polynomials which arise from base-x arithemtic, it would enhance our work in algebra teaching. These three main ideas are at the end of paragraph 2 and the beginning of paragraph 3 on page 10. You have already carried proofs of the other two propositions in the newsletter.

Thanks again,

Bruce Ewen.

November 22, 1968

Dear Sir:

I definitely agree with Mr. Bobbitt's criticism of the Mathematics 12 Departmental exams. In addition to his complaints, I would ask why the multiple-choice form of question is used so extensively. In many instances this type of question becomes a 'trick question' rather than a test of a mathematical topic.

I hope that a complaint based on Mr. Bobbitt's letter will be made to the Department of Education by the BCAMT and that this complaint will also question the format of the exam.

Yours truly,

J. B. Wright

BOOK REVIEWS

BOOK REVIEW - OF A SORT

'. . . It is known that the 360 came from the Babylonians; where they got it is not known. Wherever it came from, 360 is a primitive monstrosity undeserving of mathematical survival. It should have been pitched to the undiscriminating dodo or into the back-wash of Noah's Babylonian ark when the decimal system of numeration was invented, if any aritficial system was to be retained.

'The natural unit of angular measure is the radian . . .'

The above passage, complete with rhetoric and prejudice, seems to forestall thoughtful inquiry rather than encourage it. It provokes the following discussion.

For whom is the radian a natural unit? Why, for those accomplished in mathematics and physics in the modern setting, particularly those who need a set of formulae for motion in a circle! And do these people measure angles in radians? No, they measure in degrees, but calculate in radians.

Is there a natural (primitive, if you wish) manner of dividing a circle? Supply any elementary classroom with compasses and you can have the answer. In a very few minutes and after not too many 'ouches,' at least one youngster will discover that one can draw a circle and divide it into six parts with no bother at all.

Now, if we wished to subdivide one of these 'natural' divisions of the circle, we should choose ten as the divisor. For the same reason, the ancients would choose sixty. Why use such a system? Primitive pig-headedness?

One of the important properties of the base-ten system is the indication of divisibility by two, five or their product combination, simply by examining the last digit of the number in question. How important would be this property if we had no convenient division algorithm! Is it possible the ancients used the base-60 system because divisibility by two, three, five, or any of their combinations, is indicated in the same manner? Moreover, approximately two-thirds of the numbers not so divisible are not divisible at all.

Not to know whence the Babylonians got 360 is not to look at the question. The subsequent invention of base-ten numeration conferred no new facility upon this system of angular measure. Modern efforts to replace the degree, such as the mil (1/1600th of a right angle, nearly 1/1000th of a radian), have been generally unacceptable.

But this is not important. What is important is that teachers and more capable students who will read mathematical literature to broaden their backgrounds should be led to personal investigation rather than addressed as recipients of dogma.

Mathematics, Queen and Servant of Science, by Eric Temple Bell (McGraw-Hill Paperbacks), is a most useful book for mathematics teachers. It elucidates a large number of topics we shall not encounter either in our training or courses of study, and presents them with ample regard for their historical settings. Quite a few passages should, like the one quoted, provoke second thoughts. I choose to think this is what the author intended.

Bruce Ewen

The Canadian Mathematical Congress publishes a Mathematical Booklist which should be of interest to all mathematics teachers. It was prepared to help students and teachers who are forming mathematical reading lists or libraries. Copies may be obtained from the

Canadian Mathematical Congress,
985 Sherbrooke Street West,
Montreal, Quebec.

Cost 9¢ plus
6¢ postage.

UNESCO Publishes Collection of Mathematics Papers

New Trends in Mathematics Teaching, Volume I, is a publication of UNESCO, the United Nations Education, Scientific and Cultural Organization. Written in both French and English, it is a collection of papers by a number of internationally known mathematicians and mathematics educators. According to its foreword, the book was prepared in close collaboration with the International Commission of Mathematical Instruction of the International Mathematical Union and is intended for use by mathematics teachers in teaching-training institutions and secondary schools. Articles that were written in French are reviewed in English, and vice versa.

This 438-page paperbound book may be purchased from the UNESCO Pulbications Center, 317 East 34th Street, New York, New York 10016.

DO NOT FORGET TO SHOW THE MU ALPHA THETA BOOKLIST TO THE LIBRARIAN IN YOUR SCHOOL.

X68-580:1f

MEMBERSHIP APPLICATION FORM

National Council of Teachers of Mathematics
1201 Sixteenth Street, N.W., Washington, D. C. 20036

I hereby apply for membership in the National Council of Teachers of Mathematics. I understand that my membership includes a subscription to either the *Mathematics Teacher* or the *Arithmetic Teacher*. (Applications for membership must be accompanied by remittance.)

I enclose
- ☐ \$5.00, Membership Dues, including one journal
- ☐ \$8.00, Membership Dues, including both journals
- ☐ \$1.50, Student Membership, including one journal *
- ☐ \$2.50, Student Membership, including both journals *

(See membership provisions on adjoining page.)
(Student memberships must be certified.) *
(For each journal desired, add 50¢ for foreign mailing, 25¢ for Canadian mailing.)

I desire the following journal(s): ☐ *Mathematics Teacher* ☐ *Arithmetic Teacher* / ☐ New membership ☐ Renewal

I also enclose ☐ \$ .50 Additional fee for an individual subscription to the *Mathematics Student Journal.*

(Make checks payable to the National Council of Teachers of Mathematics)

Name (Please print) ____________________

Street Address ____________________

City ____________ State ____________ ZIP code ____________

* Student membership certified by ____________________