Area of Learning: Mathematics	**Pre-calculus 12**
Big Ideas	Elaborations

• Using **inverses** is the foundation of solving equations and can be extended to relationships between functions.	• **inverses:**
	  ○ *undo* the operations within an expression or function to reduce the expression to an identity (e.g., x =)
	• *Sample questions to support inquiry with students:*
	  ○ How can the inverse help to solve an equation?
	  ○ How is solving an equation related to identifying the specific input for a function, given a specific output?
	  ○ How are exponential and logarithmic functions related?
	  ○ How are the laws of exponents connected to the laws of logarithms?
	  ○ What are some other examples of inversely related functions?
	  ○ How are inverses related graphically, and why?
	  ○ How is solving an exponential equation similar to solving a trigonometric equation?
	  ○ How are inverse operations related to solving a polynomial equation by factoring?
	  ○ What is the value of using trigonometric identities to find equivalent expressions?
	  ○ Why do some equations have extraneous roots and other equations do not?

• Understanding the characteristics of families of **functions** allows us to model and understand relationships and to build connections between classes of functions.	• **functions:**
	• *Sample questions to support inquiry with students:*
	  ○ How do we decide which kind of function to use to model a given problem?
	  ○ What do functions and relations look like beyond the visible axes?
	  ○ A set of data looks like a parabola, but it is not. What function could be used to model this data?
	  ○ What does the number of zeros tell us about a function?
	  ○ What connections do we see within the characteristics of a particular class of function?

• **Transformations** of shapes extend to functions and relations in all of their representations.	• **Transformations:** • *Sample questions to support inquiry with students:* ○ How can we tell whether a transformation will have invariant points?

	○ Under what circumstances will different transformations produce the same result? ○ How do graphical transformations affect the tables of values? ○ How does a transformation affect a point found at the origin as compared to a point on an axis or a point in one of the four quadrants? ○ How can a rational function of the form $y = \frac{ax+b}{cx+d}$ be considered as a transformation of the reciprocal function $y = \frac{1}{x}$?

Learning Standards			
Curricular Competencies	Elaborations	Content	Elaborations
Students are expected to do the following: Reasoning and modelling • Develop **thinking strategies** to solve puzzles and play games • Explore, **analyze**, and apply mathematical ideas using **reason**, **technology**, and **other tools** • **Estimate reasonably** and demonstrate **fluent, flexible, and strategic thinking** about number • **Model** with mathematics in **situational context**s • **Think creatively** and with **curiosity and wonder** when exploring problems Understanding and solving • Develop, demonstrate, and apply conceptual understanding of mathematical ideas through play, story, **inquiry**, and problem solving • **Visualize** to explore and illustrate	• **thinking strategies:** ○ using reason to determine winning strategies ○ generalizing and extending • **analyze:** ○ examine the structure of and connections between mathematical ideas (e.g., exponential functions to geometric sequences) • **reason**: ○ inductive and deductive reasoning ○ predictions, generalizations, conclusions drawn from experiences (e.g., with puzzles, games, and coding) • **technology:** ○ graphing technology, dynamic geometry, calculators, virtual manipulatives, concept-based apps ○ can be used to for a wide variety of purposes, including: – exploring and demonstrating	*Students are expected to know the following:* • **transformations** of functions and relations • **exponential** functions and equations • **geometric** sequences and series • **logarithms:** operations, functions, and equations • **polynomial** functions and equations • **rational** functions • **trigonometry:** functions, equations, and identities	• **transformations:** ○ of graphs and equations of parent functions and relations (e.g., absolute value, radical, reciprocal, conics, exponential, logarithmic, trigonometric) ○ vertical and horizontal translations, stretches, and reflections ○ inverses: graphs and equations ○ extension: – recognizing composed functions (e.g., $y =$) – operations on functions • **exponential:** ○ graphing, including transformations ○ solving equations with same base and with different bases, including base *e* ○ solving problems in situational contexts • **geometric:** ○ common ratio, first term, general

mathematical concepts and relationships • Apply **flexible and strategic approaches** to **solve problems** • Solve problems with **persistence and a positive disposition** • Engage in problem-solving experiences **connected** with place, story, cultural practices, and perspectives relevant to local First Peoples communities, the local community, and other cultures Communicating and representing • **Explain and justify** mathematical ideas and **decisions** in **many ways** • **Represent** mathematical ideas in concrete, pictorial, and symbolic forms • Use mathematical vocabulary and language to contribute to **discussions** in the classroom • Take risks when offering ideas in classroom **discourse** Connecting and reflecting • **Reflect** on mathematical thinking • **Connect mathematical concepts** with each other, other areas, and personal interests • Use **mistakes** as **opportunities to advance learning** • **Incorporate** First Peoples worldviews,	mathematical relationships – organizing and displaying data – generating and testing inductive conjectures – mathematical modelling • **other tools:** ○ manipulatives such as algebra tiles and other concrete materials • **Estimate reasonably:** ○ be able to defend the reasonableness of an estimated value or a solution to a problem or equation (e.g., the zeros of a graphed polynomial function) • **fluent, flexible and strategic thinking:** ○ includes: – using known facts and benchmarks, partitioning, applying whole number strategies to rational numbers and algebraic expressions – choosing from different ways to think of a number or operation (e.g., Which will be the most strategic or efficient?) • **Model:** ○ use mathematical concepts and tools to solve problems and make decisions (e.g., in real-life and/or abstract scenarios) ○ take a complex, essentially non-		term ○ geometric sequences connecting to exponential functions ○ infinite geometric series ○ sigma notation • **logarithms:** ○ applying laws of logarithms ○ evaluating with different bases ○ using common and natural logarithms ○ exploring inverse of exponential ○ graphing, including transformations ○ solving equations with same base and with different bases ○ solving problems in situational contexts • **polynomial:** ○ factoring, including the factor theorem and the remainder theorem ○ graphing and the characteristics of a graph (e.g., degree, extrema, zeros, end-behaviour) ○ solving equations algebraically and graphically • **rational:** ○ characteristics of graphs, including asymptotes, intercepts, point discontinuities, domain, end-behaviour • **trigonometry:** ○ examining angles in standard position

perspectives, **knowledge**, and **practices** to make connections with mathematical concepts	mathematical scenario and figure out what mathematical concepts and tools are needed to make sense of it
	• **situational contexts:**
	  ○ including real-life scenarios and open-ended challenges that connect mathematics with everyday life
	• **Think creatively:**
	  ○ by being open to trying different strategies
	  ○ refers to creative and innovative mathematical thinking rather than to representing math in a creative way, such as through art or music
	• **curiosity and wonder:**
	  ○ asking questions to further understanding or to open other avenues of investigation
	• **inquiry:**
	  ○ includes structured, guided, and open inquiry
	  ○ noticing and wondering
	  ○ determining what is needed to make sense of and solve problems
	• **Visualize:**
	  ○ create and use mental images to support understanding
	  ○ Visualization can be supported using dynamic materials (e.g., graphical relationships and simulations),
			in both radians and degrees
			○ exploring unit circle, reference and coterminal angles, special angles
			○ graphing primary trigonometric functions, including transformations and characteristics
			○ solving first- and second-degree equations (over restricted domains and all real numbers)
			○ solving problems in situational contexts
			○ using identities to reduce complexity in expressions and solve equations (e.g., Pythagorean, quotient, double angle, reciprocal, sum and difference)

	concrete materials, drawings, and diagrams.
	• **flexible and strategic approaches:**
	  ○ deciding which mathematical tools to use to solve a problem
	  ○ choosing an effective strategy to solve a problem (e.g., guess and check, model, solve a simpler problem, use a chart, use diagrams, role-play)
	• **solve problems:**
	  ○ interpret a situation to identify a problem
	  ○ apply mathematics to solve the problem
	  ○ analyze and evaluate the solution in terms of the initial context
	  ○ repeat this cycle until a solution makes sense
	• **persistence and a positive disposition:**
	  ○ not giving up when facing a challenge
	  ○ problem solving with vigour and determination
	• **connected:**
	  ○ through daily activities, local and traditional practices, popular media and news events, cross-curricular integration
	  ○ by posing and solving problems or asking questions about place, stories, and cultural practices		

	• **Explain and justify:**
	  ○ use mathematical arguments to convince
	  ○ includes anticipating consequences
	• **decisions:**
	  ○ Have students explore which of two scenarios they would choose and then defend their choice.
	• **many ways:**
	  ○ including oral, written, visual, use of technology
	• **Represent:**
	  ○ using models, tables, graphs, words, numbers, symbols
	  ○ connecting meanings among various representations
	• **discussions:**
	  ○ partner talks, small-group discussions, teacher-student conferences
	• **discourse:**
	  ○ is valuable for deepening understanding of concepts
	  ○ can help clarify students' thinking, even if they are not sure about an idea or have misconceptions
	• **Reflect:**
	  ○ share the mathematical thinking of self and others, including evaluating strategies and solutions, extending,		

	posing new problems and questions • **Connect mathematical concepts:** ○ to develop a sense of how mathematics helps us understand ourselves and the world around us (e.g., daily activities, local and traditional practices, popular media and news events, social justice, cross-curricular integration) • **mistakes:** ○ range from calculation errors to misconceptions • **opportunities to advance learning:** ○ by: – analyzing errors to discover misunderstandings – making adjustments in further attempts – identifying not only mistakes but also parts of a solution that are correct • **Incorporate:** ○ by: – collaborating with Elders and knowledge keepers among local First Peoples – exploring the First Peoples Principles of Learning (http://www.fnesc.ca/wp/wp-content/uploads/2015/09/PUB-		

	LFP-POSTER-Principles-of-Learning-First-Peoples-poster-11x17.pdf; e.g., Learning is holistic, reflexive, reflective, experiential, and relational [focused on connectedness, on reciprocal relationships, and a sense of place]; Learning involves patience and time) – making explicit connections with learning mathematics – exploring cultural practices and knowledge of local First Peoples and identifying mathematical connections • **knowledge:** ○ local knowledge and cultural practices that are appropriate to share and that are non-appropriated • **practices:** ○ Bishop's cultural practices: counting, measuring, locating, designing, playing, explaining (http://www.csus.edu/indiv/o/oreyd/ACP.htm_files/abishop.htm) ○ Aboriginal Education Resources (www.aboriginaleducation.ca) ○ *Teaching Mathematics in a First Nations Context*, FNESC (http://www.fnesc.ca/resources/math		

Comment [mw1]: Carpe Diem: Possible to embed link in FPPL, or does URL have to be visible?

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