Area of Learning: Mathematics	**Geometry 12**
Big Ideas	Elaborations

• **Diagrams** are fundamental to investigating, communicating, and discovering properties and relations in geometry.	• **Diagrams:**
	• *Sample questions to support inquiry with students:*
	  ○ How would we describe a specific geometric object to someone who cannot see it?
	  ○ What properties can we infer from a diagram?
	  ○ What behaviours can we infer from a dynamic diagram?

• Finding **invariance amidst variation** drives geometric investigation.	• **invariance amidst variation:**
	  ○ Invariance amidst variation can be more easily experienced using current technology and dynamic diagrams. For example, the sum of the angles in planar triangles is invariant no matter what forms a triangle takes.
	• *Sample questions to support inquiry with students:*
	  ○ How do we construct geometric shapes that maintain properties under variation?
	  ○ What properties change and stay the same when we vary a square, parallelogram, triangle, and so on?
	  ○ How can the Pythagorean theorem be restated in terms of variance and invariance?

• Geometry involves creating, testing, and refining **definitions**.	• **definitions:**
	  ○ are seldom the starting point in geometry
	• *Sample questions to support inquiry with students:*
	  ○ How does variation help to refine our definitions of shapes?
	  ○ How would we define a square (or a circle) in different ways? When would one definition be better to work with than another?
	  ○ How can the definition of a shape be used in constructing the shape?
	  ○ How can we modify a definition of a shape to define a new shape?

• The **proving process** begins with conjecturing, looking for counter-examples, and refining the conjecture, and the process may end with a written proof.	• **proving process:**
	• *Sample questions to support inquiry with students:*
	  ○ Can we make a conjecture about the diagonals of a polygon? Can we find a counter-example to our conjecture?
	  ○ How can one conjecture about a *specific* shape lead to making another more *general* conjecture about a family of shapes?
	  ○ How can we be sure that a proof is complete?
	  ○ Can we find a counter-example to a conjecture?

	○ How can different proofs bring out different understandings of a relationship?
• **Geometry** stories and applications vary across cultures and time.	• **Geometry:** ○ Geometry is more than a list of axioms and deductions. Non-Western and modern geometry is concerned with shape and space and is not always axiomatic. It is not always about producing a theorem; rather, it is about modelling mathematical and non-mathematical phenomena using geometric objects and relations. Today geometry is used in a multitude of disciplines, including animation, architecture, biology, carpentry, chemistry, medical imaging, and art. • *Sample questions to support inquiry with students:* ○ Can we find geometric relationships in local First Peoples art or culture? ○ Can we make geometric connections to story, language, or past experiences? ○ What do we notice about and how would we construct common shapes found in local First Peoples art? ○ How has the notion of "proof" changed over time and in different cultures? ○ How are geometric ideas implemented in modern professions?

Curricular Competencies	Elaborations	Content	Elaborations
Students are expected to do the following: Reasoning and modelling • Develop **thinking strategies** to solve puzzles and play games • Engage in **spatial reasoning** in a dynamic environment • Explore, **analyze**, and apply mathematical ideas using **reason**, **technology**, and **other tools** • **Estimate reasonably** and demonstrate **fluent, flexible, and strategic thinking** about number • **Model** with mathematics in **situational contexts**	• **thinking strategies:** ○ using reason to determine winning strategies ○ generalizing and extending • **spatial reasoning:** ○ being able to think about shapes (real or imagined) and mentally transform them to notice relationships • **analyze:** ○ examine the structure of and connections between geometric ideas (e.g., parallel and perpendicular lines, circle geometry, constructing tangents, transformations) • **reason:** ○ inductive and deductive reasoning	*Students are expected to know the following:* • geometric **constructions** • **parallel and perpendicular** lines: ○ **circles as tools** in constructions ○ perpendicular bisector • **circle geometry** • **constructing tangents** • transformations of 2D shapes: ○ **isometries** ○ **non-isometric transformations** • **non-Euclidean geometries**	• **constructions:** ○ angles, triangles, triangle centres, quadrilaterals • **parallel and perpendicular:** ○ angle bisector • **circles as tools:** ○ constructing equal segments, midpoints • **circle geometry:** ○ properties of chords, angles, and tangents to mobilize the proving process • **constructing tangents:** ○ lines tangent to circles, circles tangent to circles, circles tangent to three objects (e.g., points [PPP],

• **Think creatively** and with **curiosity and wonder** when exploring problems Understanding and solving • Develop, demonstrate, and apply conceptual understanding of mathematical ideas through play, story, **inquiry**, and problem solving • **Visualize** to explore and illustrate geometric concepts and relationships • Apply **flexible and strategic approaches** to **solve problems** • Solve problems with **persistence and a positive disposition** • Engage in problem-solving experiences **connected** with place, story, cultural practices, and perspectives relevant to local First Peoples communities, the local community, and other cultures Communicating and representing • **Explain, justify,** and evaluate geometric ideas and **decisions** in **many ways** • **Represent** mathematical ideas in concrete, pictorial, and symbolic forms • Use geometric vocabulary and language to contribute to **discussions** in the classroom • Take risks when offering ideas in classroom **discourse**	○ predictions, generalizations, conclusions drawn from experiences (e.g., with puzzles, games, and coding) • **technology:** ○ graphing technology, dynamic geometry, calculators, virtual manipulatives, concept-based apps ○ can be used for a wide variety of purposes, including: – exploring and demonstrating geometrical relationships – organizing and displaying data – generating and testing inductive conjectures – mathematical modelling • **other tools:** ○ paper and scissors, straightedge and compass, ruler, and other concrete materials • **Estimate reasonably:** ○ be able to defend the reasonableness of an estimated value or a solution to a problem or equation (e.g., congruencies, angles, lengths) • **fluent, flexible, and strategic thinking:** ○ being able to generate a family of shapes and apply characteristics across the family • **Model:** ○ use mathematical concepts and tools to solve problems and make		three lines [LLL]) • **isometries:** ○ transformations that maintain congruence (translations, rotations, reflections) ○ composition of isometries ○ tessellations • **non-isometric transformations:** ○ dilations and shear ○ topology • **non-Euclidean geometries:** ○ perspective, spherical, Taxicab, hyperbolic ○ tessellations

Connecting and reflecting • **Reflect** on geometric thinking • **Connect mathematical concepts** with each other, other areas, and personal interests • Use **mistakes** as **opportunities to advance learning** • **Incorporate** First Peoples worldviews, perspectives, **knowledge**, and **practices** to make connections with mathematical concepts	decisions (e.g., in real-life and/or abstract scenarios) ○ take a complex, essentially non-mathematical scenario and figure out what mathematical concepts and tools are needed to make sense of it • **situational contexts:** ○ including real-life scenarios and open-ended challenges that connect mathematics with everyday life • **Think creatively:** ○ by being open to trying different strategies ○ refers to creative and innovative mathematical thinking rather than to representing math in a creative way, such as through art or music • **curiosity and wonder:** ○ asking questions to further understanding or to open other avenues of investigation • **inquiry:** ○ includes structured, guided, and open inquiry ○ noticing and wondering ○ determining what is needed to make sense of and solve problems • **Visualize:** ○ create and use mental images to support understanding ○ Visualization can be supported using dynamic materials (e.g., graphical relationships and simulations),		

	concrete materials, drawings, and diagrams. • **flexible and strategic approaches:** ○ deciding which mathematical tools to use to solve a problem ○ choosing an effective strategy to solve a problem (e.g., guess and check, model, solve a simpler problem, use a chart, use diagrams, role-play) • **solve problems:** ○ interpret a situation to identify a problem ○ apply mathematics to solve the problem ○ analyze and evaluate the solution in terms of the initial context ○ repeat this cycle until a solution makes sense • **persistence and a positive disposition:** ○ not giving up when facing a challenge ○ problem solving with vigour and determination • **connected:** ○ through daily activities, local and traditional practices, popular media and news events, cross-curricular integration ○ by posing and solving problems or asking questions about place, stories, and cultural practices • **Explain and justify:**		

	○ use mathematical arguments to convince ○ includes anticipating consequences • **decisions**: ○ Have students explore which of two scenarios they would choose and then defend their choice. • **many ways:** ○ including oral, written, visual, gestures and use of technology ○ communicating effectively according to what is being communicated and to whom • **Represent:** ○ concretely, diagrammatically, symbolically, including using models, tables, graphs, words, numbers, symbols • **discussions:** ○ partner talks, small-group discussions, teacher-student conferences • **discourse:** ○ is valuable for deepening understanding of concepts ○ can help clarify students' thinking, even if they are not sure about an idea or have misconceptions • **Reflect:** ○ share the geometric thinking of self and others, including evaluating strategies and solutions, finding counter-examples, extending, posing		

	new problems and questions, proving results
	• **Connect mathematical concepts:**
	  ○ to develop a sense of how mathematics helps us understand ourselves and the world around us (e.g., daily activities, local and traditional practices, popular media and news events, social justice, cross-curricular integration)
	• **mistakes:**
	  ○ range from calculation errors to misconceptions
	• **opportunities to advance learning:**
	  ○ by:
	    – analyzing errors to discover misunderstandings
	    – making adjustments in further attempts
	    – identifying not only mistakes but also parts of a solution that are correct
	• **Incorporate:**
	  ○ by:
	    – collaborating with Elders and knowledge keepers among local First Peoples
	    – exploring the First Peoples Principles of Learning (http://www.fnesc.ca/wp/wp-content/uploads/2015/09/PUB-		

	LFP-POSTER-Principles-of-Learning-First-Peoples-poster-11x17.pdf; e.g., Learning is holistic, reflexive, reflective, experiential, and relational [focused on connectedness, on reciprocal relationships, and a sense of place]; Learning involves patience and time) – making explicit connections with learning mathematics – exploring cultural practices and knowledge of local First Peoples and identifying mathematical connections • **knowledge:** ○ local knowledge and cultural practices that are appropriate to share and that are non-appropriated • **practices:** ○ Bishop's cultural practices: counting, measuring, locating, designing, playing, explaining (http://www.csus.edu/indiv/o/oreyd/ACP.htm_files/abishop.htm) ○ Aboriginal Education Resources (www.aboriginaleducation.ca) ○ *Teaching Mathematics in a First Nations Context*, FNESC (http://www.fnesc.ca/resources/mat		

Comment [ANE1]: Carpe Diem: Please add hyperlink as per other documents.

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