Area of Learning: Mathematics	**Calculus 12**
Big Ideas	Elaborations

• The **concept of a limit** is foundational to calculus.	• **concept of a limit:**
	  ○ Differentiation and integration are defined using limits.
	• *Sample questions to support inquiry with students:*
	  ○ Why is a limit useful?
	  ○ How can we use historical examples (e.g., Achilles and the tortoise) to describe a limit?

• Differential calculus develops the concept of **instantaneous rate of change**.	• **instantaneous rate of change:**
	  ○ developing rate of change from average to instantaneous
	• *Sample questions to support inquiry with students:*
	  ○ How can a rate of change be instantaneous?
	  ○ When do we use rate of change?

• Integral calculus develops the concept of determining a product involving a **continuously changing** quantity over an interval.	• **continuously changing:**
	  ○ area (height x width) under a curve where the height of the region is changing; volume of a solid (area x length) where cross-sectional area is changing; work (force x distance) where force is changing
	  ○ Finding these products requires finding an infinite sum.
	• *Sample questions to support inquiry with students:*
	  ○ What is the value of using rectangles to approximate the area under a curve?
	  ○ Why is the fundamental theorem of calculus so fundamental?

• Derivatives and integrals are **inversely related.**	• **inversely related:**
	  ○ The fundamental theorem of calculus describes the relationship between integrals and antiderivatives.
	• *Sample questions to support inquiry with students:*
	  ○ How are derivatives and integrals related?
	  ○ Why are antiderivatives important?
	  ○ What is the difference between an antiderivative and an integral?

Learning Standards			
Curricular Competencies	Elaborations	Content	Elaborations
Students are expected to do the following:	• **thinking strategies:**	*Students are expected to know the*	• **functions:**

Reasoning and modelling • Develop **thinking strategies** to solve puzzles and play games • Explore, **analyze**, and apply mathematical ideas using **reason**, **technology**, and **other tools** • **Estimate reasonably** and demonstrate **fluent, flexible, and strategic thinking** about number • **Model** with mathematics in **situational contexts** • **Think creatively** and with **curiosity and wonder** when exploring problems Understanding and solving • Develop, demonstrate, and apply conceptual understanding of mathematical ideas through play, story, **inquiry**, and problem solving • **Visualize** to explore and illustrate mathematical concepts and relationships • Apply **flexible and strategic approaches** to **solve problems** • Solve problems with **persistence and a positive disposition** • Engage in problem-solving experiences **connected** with place, story, cultural practices, and perspectives relevant to local First Peoples communities, the local community, and other cultures Communicating and representing	○ using reason to determine winning strategies ○ generalizing and extending • **analyze:** ○ examine the structure of and connections between mathematical ideas (e.g., limits, derivatives, integrals) • **reason**: ○ inductive and deductive reasoning ○ predictions, generalizations, conclusions drawn from experiences (e.g., in puzzles, games, coding) • **technology:** ○ graphing technology, dynamic geometry, calculators, virtual manipulatives, concept-based apps ○ can be used for a wide variety of purposes, including: – exploring and demonstrating mathematical relationships – organizing and displaying data – generating and testing inductive conjectures – mathematical modelling • **other tools:** ○ manipulatives such as algebra tiles and other concrete materials • **Estimate reasonably:** ○ be able to defend the reasonableness of an estimate across mathematical contexts • **fluent, flexible, and strategic thinking:**	*following:* • **functions** and graphs • **limits:** ○ left and right limits ○ limits to infinity ○ continuity • **differentiation:** ○ **rate of change** ○ **differentiation rules** ○ higher order, implicit ○ **applications** • **integration:** ○ **approximations** ○ fundamental theorem of calculus ○ **methods of integration** ○ **applications**	○ parent functions from Pre-Calculus 12 ○ piecewise functions ○ inverse trigonometric functions • **limits:** ○ from table of values, graphically, and algebraically ○ one-sided versus two-sided ○ end behaviour ○ intermediate value theorem • **differentiation:** ○ history ○ definition of derivative ○ notation • **rate of change:** ○ average versus instantaneous ○ slope of secant and tangent lines • **differentiation rules:** ○ power, product; quotient and chain ○ transcendental functions: logarithmic, exponential, trigonometric • **applications:** ○ relating graph of *f(x)* to *f'(x)* and *f"(x)* ○ increasing/decreasing, concavity ○ differentiability, mean value theorem ○ Newton's method ○ problems in contextual situations, including related rates and optimization problems • **integration:** ○ definition of an integral ○ notation

• **Explain and justify** mathematical ideas and **decisions** in **many ways** • **Represent** mathematical ideas in concrete, pictorial, and symbolic forms • Use mathematical vocabulary and language to contribute to **discussions** in the classroom • Take risks when offering ideas in classroom **discourse** Connecting and reflecting • **Reflect** on mathematical thinking • **Connect mathematical concepts** with each other, other areas, and personal interests • Use **mistakes** as **opportunities to advance learning** • **Incorporate** First Peoples worldviews, perspectives, **knowledge**, and **practices** to make connections with mathematical concepts	○ includes: – using known facts and benchmarks, partitioning, applying number strategies to approximate limits, derivatives, and integrals – choosing from different ways to think of a number or operation (e.g., Which will be the most strategic or efficient?) • **Model:** ○ use mathematical concepts and tools to solve problems and make decisions (e.g., in real-life and/or abstract scenarios) ○ take a complex, essentially non-mathematical scenario and figure out what mathematical concepts and tools are needed to make sense of it • **situational contexts:** ○ including real-life scenarios and open-ended challenges that connect mathematics with everyday life • **Think creatively:** ○ by being open to trying different strategies ○ refers to creative and innovative mathematical thinking rather than to representing math in a creative way, such as through art or music • **curiosity and wonder:** ○ asking questions to further understanding or to open other		○ definite and indefinite • **approximations:** ○ Riemann sum, rectangle approximation method, trapezoidal method • **methods of integration:** ○ antiderivatives of functions ○ substitution ○ by parts • **applications:** ○ area under a curve, volume of solids, average value of functions ○ differential equations ○ initial value problems ○ slope fields

	avenues of investigation • **inquiry:** ○ includes structured, guided, and open inquiry ○ noticing and wondering ○ determining what is needed to make sense of and solve problems • **Visualize:** ○ create and use mental images to support understanding ○ Visualization can be supported using dynamic materials (e.g., graphical relationships and simulations), concrete materials, drawings, and diagrams. • **flexible and strategic approaches:** ○ deciding which mathematical tools to use to solve a problem ○ choosing an effective strategy to solve a problem (e.g., guess and check, model, solve a simpler problem, use a chart, use diagrams, role-play) • **solve problems:** ○ interpret a situation to identify a problem ○ apply mathematics to solve the problem ○ analyze and evaluate the solution in terms of the initial context ○ repeat this cycle until a solution makes sense • **persistence and a positive disposition:**		

	○ not giving up when facing a challenge ○ problem solving with vigour and determination • **connected:** ○ through daily activities, local and traditional practices, popular media and news events, cross-curricular integration ○ by posing and solving problems or asking questions about place, stories, and cultural practices • **Explain and justify:** ○ using mathematical arguments to convince ○ includes anticipating consequences • **decisions:** ○ Have students explore which of two scenarios they would choose and then defend their choice. • **many ways:** ○ including oral, written, visual, use of technology ○ communicating effectively according to what is being communicated and to whom • **Represent:** ○ using models, tables, graphs, words, numbers, symbols ○ connecting meanings among various representations • **discussions:** ○ partner talks, small-group discussions, teacher-student		

	conferences
	• **discourse:**
	  ○ is valuable for deepening understanding of concepts
	  ○ can help clarify students’ thinking, even if they are not sure about an idea or have misconceptions
	• **Reflect:**
	  ○ share the mathematical thinking of self and others, including evaluating strategies and solutions, extending, posing new problems and questions
	• **Connect mathematical concepts:**
	  ○ to develop a sense of how mathematics helps us understand ourselves and the world around us (e.g., daily activities, local and traditional practices, popular media and news events, social justice, cross-curricular integration)
	• **mistakes:**
	  ○ range from calculation errors to misconceptions
	• **opportunities to advance learning**:
	  ○ by:
	    – analyzing errors to discover misunderstandings
	    – making adjustments in further attempts
	    – identifying not only mistakes but also parts of a solution that are correct
	• **Incorporate:**		

	◦ by: – collaborating with Elders and knowledge keepers among local First Peoples – exploring the First Peoples Principles of Learning (http://www.fnesc.ca/wp/wp-content/uploads/2015/09/PUB-LFP-POSTER-Principles-of-Learning-First-Peoples-poster-11x17.pdf; e.g., Learning is holistic, reflexive, reflective, experiential, and relational [focused on connectedness, on reciprocal relationships, and a sense of place]; Learning involves patience and time) – making explicit connections with learning mathematics – exploring cultural practices and knowledge of local First Peoples and identifying mathematical connections • **knowledge:** ◦ local knowledge and cultural practices that are appropriate to share and that are non-appropriated • **practices:** ◦ Bishop's cultural practices: counting, measuring, locating, designing, playing, explaining (http://www.csus.edu/indiv/o/oreyd/ACP.htm_files/abishop.htm)		

Comment [mw1]: Carpe Diem: Possible to embed link in FPPL or does URL have to be visible?

	◦ Aboriginal Education Resources (www.aboriginaleducation.ca) ◦ *Teaching Mathematics in a First Nations Context*, FNESC (http://www.fnesc.ca/resources/math-first-peoples/)		