Area of Learning: Mathematics	**Pre-calculus 11**
Big Ideas	Elaborations

• Algebra allows us to **generalize** relationships through abstract thinking.	• **generalize:**
	• *Sample questions to support inquiry with students:*
	  ○ After solving a problem, can we extend it? Can we generalize it?
	  ○ How can we take a contextualized problem and turn it into a mathematical problem that can be solved?
	  ○ How do we tell if a mathematical solution is reasonable?
	  ○ Where can errors occur when solving a contextualized problem?
	  ○ What are the similarities and differences between quadratic functions and linear functions? How are they connected?
	  ○ What do we notice about the rate of change in a quadratic function?
	  ○ How do the strategies for solving linear equations extend to solving quadratic, radical, or rational equations?
	  ○ What is the connection between domain and extraneous roots?

• The meanings of, and **connections** between, operations extend to powers, radicals, and polynomials.	• **connections:**
	• *Sample questions to support inquiry with students:*
	  ○ How are the different operations (+, -, x, ÷, exponents, roots) connected?
	  ○ What are the similarities and differences between multiplication of numbers, powers, radicals, polynomials, and rational expressions?
	  ○ How can we verify that we have factored a trinomial correctly?
	  ○ How can visualization support algebraic thinking?
	  ○ How can patterns in numbers lead to algebraic generalizations?
	  ○ When would we choose to represent a number with a radical rather than a rational exponent?
	  ○ How do strategies for factoring $x^2 + bx + c$ extend to $ax^2 + bx + c, a \neq 1$

	○ How do operations on rational numbers extend to operations with rational expressions?
• Quadratic **relationships** are prevalent in the world around us.	• **relationships:** • *Sample questions to support inquiry with students:* ○ What are some examples of quadratic relationships in the world around us, and what are the similarities and differences between these? ○ Why are quadratic relationships so prevalent in the world around us? ○ How does the predictable pattern of linear functions extend to quadratic functions? ○ Why is the shape of a quadratic function called a parabola? ○ How can we decide which form of a quadratic function to use for a given problem? ○ What effect does each term of a quadratic function have on its graph?
• Trigonometry involves using **proportional reasoning** to solve **indirect measurement** problems.	• **proportional reasoning:** ○ comparisons of relative size or scale instead of numerical difference • **indirect measurement:** ○ using measurable values to calculate immeasurable values (e.g., calculating the width of a river using the distance between two points on one shore and an angle to a point on the other shore) • *Sample questions to support inquiry with students:* ○ How is the cosine law related to the Pythagorean theorem? ○ How can we use right triangles to find a rule for solving non-right triangles? ○ How do we decide when to use the sine law or cosine law? ○ What would it mean for an angle to have a negative measure? Identify a context for making sense of a negative angle.

Comment [mw1]: Carpe Diem: can we make the questions pop up with both elaborations?

Curricular Competencies	Elaborations	Content	Elaborations
Students are expected to do the following: Reasoning and modelling • Develop **thinking strategies** to solve puzzles and play games	• **thinking strategies:** ○ using reason to determine winning strategies ○ generalizing and extending • **analyze:**	*Students are expected to know the following:* • **real number** system • **powers** with rational exponents • **radical** operations and equations	• **real number:** ○ classification • **powers:** ○ positive and negative rational exponents

• Explore, **analyze**, and apply mathematical ideas using **reason**, **technology**, and **other tools** • **Estimate reasonably** and demonstrate **fluent, flexible, and strategic thinking** about number • **Model** with mathematics in **situational context**s • **Think creatively** and with **curiosity and wonder** when exploring problems Understanding and solving • Develop, demonstrate, and apply conceptual understanding of mathematical ideas through play, story, **inquiry**, and problem solving • **Visualize** to explore and illustrate mathematical concepts and relationships • Apply **flexible and strategic approaches** to **solve problems** • Solve problems with **persistence and a positive disposition** • Engage in problem-solving experiences **connected** with place, story, cultural practices, and perspectives relevant to local First Peoples communities, the local community, and other cultures Communicating and representing • **Explain and justify** mathematical ideas	○ examine the structure of and connections between mathematical ideas (e.g., trinomial factoring, roots of quadratic equations) • **reason:** ○ inductive and deductive reasoning ○ predictions, generalizations, conclusions drawn from experiences (e.g., with puzzles, games, and coding) • **technology:** ○ graphing technology, dynamic geometry, calculators, virtual manipulatives, concept-based apps ○ can be used for a wide variety of purposes, including: – exploring and demonstrating mathematical relationships – organizing and displaying data – generating and testing inductive conjectures – mathematical modelling • **other tools:** ○ manipulatives such as algebra tiles and other concrete materials • **Estimate reasonably:** ○ be able to defend the reasonableness of an estimated value or a solution to a problem or equation (e.g., the zeros of a graphed polynomial function) • **fluent, flexible and strategic thinking:**	• polynomial **factoring** • **rational** expressions and equations • **quadratic** functions and equations • linear and quadratic **inequalities** • **trigonometry:** non-right triangles and angles in standard position • **financial literacy:** compound interest, investments, loans	○ exponent laws ○ evaluation using order of operations ○ numerical and variable bases • **radical:** ○ simplifying radicals ○ ordering a set of irrational numbers ○ performing operations with radicals ○ solving simple (one radical only) equations algebraically and graphically ○ identifying domain restrictions and extraneous roots of radical equations • **factoring:** ○ greatest common factor of a polynomial ○ trinomials of the form $ax^2 + bx + c$ ○ difference of squares of the form $a^2x^2 - b^2y^2$ ○ may extend to $a\big(f(x)\big)^2 + b\big(f(x)\big) + c$, $a^2\big(f(x)\big)^2 - b^2\big(f(x)\big)^2$ • **rational:** ○ simplifying and applying operations to rational expressions ○ identifying non-permissible values ○ solving equations and identifying any extraneous roots • **quadratic:** ○ identifying characteristics of graphs (including domain and range,

and **decisions** in **many ways** • **Represent** mathematical ideas in concrete, pictorial, and symbolic forms • Use mathematical vocabulary and language to contribute to **discussions** in the classroom • Take risks when offering ideas in classroom **discourse** Connecting and reflecting • **Reflect** on mathematical thinking • **Connect mathematical concepts** with each other, with other areas, and with personal interests • Use **mistakes** as **opportunities to advance learning** • **Incorporate** First Peoples worldviews, perspectives, **knowledge**, and **practices** to make connections with mathematical concepts	○ includes: – using known facts and benchmarks, partitioning, applying whole number strategies to rational numbers and algebraic expressions – choosing from different ways to think of a number or operation (e.g., Which will be the most strategic or efficient?) • **Model:** ○ use mathematical concepts and tools to solve problems and make decisions (e.g., in real-life and/or abstract scenarios) ○ take a complex, essentially non-mathematical scenario and figure out what mathematical concepts and tools are needed to make sense of it • **situational contexts**: ○ including real-life scenarios and open-ended challenges that connect mathematics with everyday life • **Think creatively:** ○ by being open to trying different strategies ○ refers to creative and innovative mathematical thinking rather than to representing math in a creative way, such as through art or music		intercepts, vertex, symmetry), multiple forms, function notation, extrema ○ exploring transformations ○ solving equations (e.g., factoring, quadratic formula, completing the square, graphing, square root method) ○ connecting equation-solving strategies ○ connecting equations with functions ○ solving problems in context • **inequalities:** ○ single variable (e.g., $3x - 7 \leq -4$, $x^2 - 5x + 6 > 0$) ○ domain and range restrictions from problems in situational contexts ○ sign analysis: identifying intervals where a function is positive, negative, or zero ○ symbolic notation for inequality statements, including interval notation • **trigonometry:** ○ use of sine and cosine laws to solve non-right triangles, including ambiguous cases ○ contextual and non-contextual problems ○ angles in standard position:

	• **curiosity and wonder:**
	  ○ asking questions to further understanding or to open other avenues of investigation
	• **inquiry:**
	  ○ includes structured, guided, and open inquiry
	  ○ noticing and wondering
	  ○ determining what is needed to make sense of and solve problems
	• **Visualize:**
	  ○ create and use mental images to support understanding
	  ○ Visualization can be supported using dynamic materials (e.g., graphical relationships and simulations), concrete materials, drawings, and diagrams.
	• **flexible and strategic approaches:**
	  ○ deciding which mathematical tools to use to solve a problem
	  ○ choosing an effective strategy to solve a problem (e.g., guess and check, model, solve a simpler problem, use a chart, use diagrams, role-play)
	• **solve problems:**
	  ○ interpret a situation to identify a problem
	  ○ apply mathematics to solve the problem
			– degrees
			– special angles, as connected with the 30-60-90 and 45-45-90 triangles
			○ unit circle
			○ reference and coterminal angles
			○ terminal arm
			○ trigonometric ratios
			○ simple trigonometric equations
			• **financial literacy:**
			○ compound interest
			○ introduction to investments/loans with regular payments, using technology
			○ buy/lease

	○ analyze and evaluate the solution in terms of the initial context ○ repeat this cycle until a solution makes sense • **persistence and a positive disposition:** ○ not giving up when facing a challenge ○ problem solving with vigour and determination • **connected:** ○ through daily activities, local and traditional practices, popular media and news events, cross-curricular integration ○ by posing and solving problems or asking questions about place, stories, and cultural practices • **Explain and justify:** ○ use mathematical arguments to convince ○ includes anticipating consequences • **decisions:** ○ Have students explore which of two scenarios they would choose and then defend their choice. • **many ways:** ○ including oral, written, visual, use of technology ○ communicating effectively according to what is being communicated and to whom		

	• **Represent:**
	  ○ using models, tables, graphs, words, numbers, symbols
	  ○ connecting meanings among various representations
	• **discussions:**
	  ○ partner talks, small-group discussions, teacher-student conferences
	• **discourse:**
	  ○ is valuable for deepening understanding of concepts
	  ○ can help clarify students' thinking, even if they are not sure about an idea or have misconceptions
	• **Reflect:**
	  ○ share the mathematical thinking of self and others, including evaluating strategies and solutions, extending, posing new problems and questions
	• **Connect mathematical concepts:**
	  ○ to develop a sense of how mathematics helps us understand ourselves and the world around us (e.g., daily activities, local and traditional practices, popular media and news events, social justice, cross-curricular integration)
	• **mistakes:**
	  ○ range from calculation errors to misconceptions		

	• **opportunities to advance learning:**
	  ○ by:
	  – analyzing errors to discover misunderstandings
	  – making adjustments in further attempts
	  – identifying not only mistakes but also parts of a solution that are correct
	• **Incorporate:**
	  ○ by:
	  – collaborating with Elders and knowledge keepers among local First Peoples
	  – exploring the First Peoples Principles of Learning (http://www.fnesc.ca/wp/wp-content/uploads/2015/09/PUB-LFP-POSTER-Principles-of-Learning-First-Peoples-poster-11x17.pdf; e.g., Learning is holistic, reflexive, reflective, experiential, and relational [focused on connectedness, on reciprocal relationships, and a sense of place]; Learning involves patience and time)
	  – making explicit connections with learning mathematics
	  – exploring cultural practices and		

Comment [mw2]: Carpe Diem: Possible to embed link in FPPL? Or does URL have to be visible?

	knowledge of local First Peoples and identifying mathematical connections
	• **knowledge:**
	  ○ local knowledge and cultural practices that are appropriate to share and that are non-appropriated
	• **practices:**
	  ○ Bishop's cultural practices: counting, measuring, locating, designing, playing, explaining (http://www.csus.edu/indiv/o/oreyd/ACP.htm_files/abishop.htm)
	  ○ Aboriginal Education Resources (www.aboriginaleducation.ca)
	  ○ *Teaching Mathematics in a First Nations Context*, FNESC (http://www.fnesc.ca/resources/math-first-peoples/)		