Area of Learning: Mathematics	**Foundations of Mathematics 11**
Big Ideas	Elaborations

• **Similar** shapes and objects have proportional relationships that can be described, measured, and compared.	• **Similar:**
	• *Sample questions to support inquiry with students:*
	  ○ What characteristics make objects similar?
	  ○ How do the properties of 3D objects change in an enlargement or a reduction?
	  ○ How do the properties of 2D objects change in an enlargement or a reduction?

• **Optimization** informs the decision-making process in situations involving extreme values.	• **Optimization:**
	  ○ a mathematical analysis used to determine the minimum or maximum output for a given situation
	• *Sample questions to support inquiry with students:*
	  ○ Can we think of a story where a conflict can be resolved through optimization?
	  ○ How can mathematics help us make decisions regarding the best course of action?
	  ○ What factors influence the decision-making process when determining an optimal solution?
	  ○ How do graphs aid in understanding a situation that is being optimized?

• **Logical reasoning** helps us discover and describe mathematical truths.	• **Logical reasoning:**
	  ○ the process of using a strategic, systematic series of steps based on valid mathematical procedures and given statements to form a conclusion
	• *Sample questions to support inquiry with students:*
	  ○ How can logical reasoning help us deal with problems in our everyday lives?
	  ○ How does puzzle and game analysis help us in the world outside the math classroom?

• Statistical analysis allows us to notice, wonder about, and answer questions about **variation**.	• **variation:**
	  ○ occurs in observation (e.g., reaction to medications, opinions on topics, income levels, graduation rates)
	• *Sample questions to support inquiry with students:*
	  ○ How do we gather data in order to answer questions?

		○ How do we analyze data and make decisions? ○ Can we think of a story that involves variation? How would we describe the variation? ○ When analyzing data, what are some of the factors that need to be considered before making inferences?	
Learning Standards			
Curricular Competencies	Elaborations	Content	Elaborations
Students are expected to do the following: Reasoning and modelling • Develop **thinking strategies** to solve puzzles and play games • Explore, **analyze**, and apply mathematical ideas using **reason**, **technology**, and **other tools** • **Estimate reasonably** and demonstrate **fluent, flexible, and strategic thinking** about number • **Model** with mathematics in **situational context**s • **Think creatively** and with **curiosity and wonder** when exploring problems Understanding and solving • Develop, demonstrate, and apply mathematical understanding through play, story, **inquiry**, and problem solving • **Visualize** to explore and illustrate mathematical concepts and relationships • Apply **flexible and strategic approaches** to **solve problems**	• **thinking strategies:** ○ using reason to determine winning strategies ○ generalizing and extending • **analyze:** ○ examine the structure of and connections between mathematical ideas (e.g., quadratics and cubic functions, linear inequalities, optimization, financial decision making) • **reason:** ○ inductive and deductive reasoning ○ predictions, generalizations, conclusions drawn from experiences (e.g., with puzzles, games, and coding) • **technology:** ○ graphing technology, dynamic geometry, calculators, virtual manipulatives, concept-based apps ○ can be used for a wide variety of purposes, including: – exploring and demonstrating mathematical relationships	*Students are expected to know the following:* • forms of **mathematical reasoning** • **angle relationships** • **graphical analysis:** ○ **linear inequalities** ○ **quadratic functions** ○ **systems of equations** ○ **optimization** • **applications** of **statistics** • **scale models** • **financial literacy:** compound interest, investments and loans	• **mathematical reasoning:** ○ logic, conjecturing, inductive and deductive thinking, proofs, game/puzzle analysis, counter-examples • **angle relationships:** ○ properties, proofs, parallel lines, triangles and other polygons, angle constructions • **graphical analysis:** ○ using technology only • **linear inequalities:** ○ graphing of the solution region ○ slope and intercepts ○ intersection points of lines • **quadratic functions:** ○ characteristics of graphs, including end behaviour, maximum/minimum, vertex, symmetry, intercepts • **systems of equations:** ○ including linear with linear, linear with quadratic, and quadratic with quadratic • **optimization:**

• Solve problems with **persistence and a positive disposition** • Engage in problem-solving experiences **connected** with place, story, cultural practices, and perspectives relevant to local First Peoples communities, the local community, and other cultures Communicating and representing • **Explain and justify** mathematical ideas and **decisions** in **many ways** • **Represent** mathematical ideas in concrete, pictorial and symbolic forms • Use mathematical vocabulary and language to contribute to **discussions** in the classroom • Take risks when offering ideas in classroom **discourse** Connecting and reflecting • **Reflect** on mathematical thinking • **Connect mathematical concepts** with each other, other areas, and personal interests • Use **mistakes** as **opportunities to advance learning** • **Incorporate** First Peoples worldviews, perspectives, **knowledge**, and **practices** to make connections with mathematical concepts	– organizing and displaying data – generating and testing inductive conjectures – mathematical modelling • **other tools:** ○ manipulatives such as algebra tiles and other concrete materials • **Estimate reasonably:** ○ be able to defend the reasonableness of an estimated value or a solution to a problem or equation (e.g., angle size reasonableness, scale calculations and unit choice, optimal solutions) • **fluent, flexible and strategic thinking:** ○ includes: – using known facts and benchmarks, partitioning, applying whole number strategies to rational numbers and algebraic expressions – choosing from different ways to think of a number or operation (e.g., Which will be the most strategic or efficient?) • **Model:** ○ use mathematical concepts and tools to solve problems and make decisions (e.g., in real-life and/or abstract scenarios)		○ using feasible region to optimize objective function ○ maximizing profit while minimizing cost ○ maximizing area or volume while minimizing perimeter • **applications:** ○ posing a question about an observed variation, collecting and interpreting data, and answering the question • **statistics:** ○ measures of central tendency, standard deviation, confidence intervals, z-scores, distributions • **scale models:** ○ enlargements and reductions of 2D shapes and 3D objects ○ comparing the properties of similar objects (length, area, volume) ○ square-cube law • **financial literacy:** ○ compound interest ○ introduction to investments/loans with regular payments using technology ○ buy/lease

	○ take a complex, essentially non-mathematical scenario and figure out what mathematical concepts and tools are needed to make sense of it • **situational contexts:** ○ including real-life scenarios and open-ended challenges that connect mathematics with everyday life • **Think creatively:** ○ by being open to trying different strategies ○ refers to creative and innovative mathematical thinking rather than to representing math in a creative way, such as through art or music • **curiosity and wonder:** ○ asking questions to further understanding or to open other avenues of investigation • **inquiry:** ○ includes structured, guided, and open inquiry ○ noticing and wondering ○ determining what is needed to make sense of and solve problems • **Visualize:** ○ create and use mental images to support understanding ○ Visualization can be supported using dynamic materials (e.g., graphical		

	relationships and simulations), concrete materials, drawings, and diagrams.
	• **flexible and strategic approaches:**
	  ○ deciding which mathematical tools to use to solve a problem
	  ○ choosing an appropriate strategy to solve a problem (e.g., guess and check, model, solve a simpler problem, use a chart, use diagrams, role-play)
	• **solve problems:**
	  ○ interpret a situation to identify a problem
	  ○ apply mathematics to solve the problem
	  ○ analyze and evaluate the solution in terms of the initial context
	  ○ repeat this cycle until a solution makes sense
	• **persistence and a positive disposition:**
	  ○ not giving up when facing a challenge
	  ○ problem solving with vigour and determination
	• **connected:**
	  ○ through daily activities, local and traditional practices, popular media and news events, cross-curricular integration
	  ○ by posing and solving problems or		

	asking questions about place, stories, and cultural practices
	• **Explain and justify:**
	  ○ use mathematical arguments to convince
	  ○ includes anticipating consequences
	• **decisions:**
	  ○ Have students explore which of two scenarios they would choose and then defend their choice.
	• **many ways:**
	  ○ including oral, written, visual, use of technology
	  ○ communicating effectively according to what is being communicated and to whom
	• **Represent:**
	  ○ using models, tables, graphs, words, numbers, symbols
	  ○ connecting meanings among various representations
	• **discussions:**
	  ○ partner talks, small-group discussions, teacher-student conferences
	• **discourse:**
	  ○ is valuable for deepening understanding of concepts
	  ○ can help clarify students' thinking, even if they are not sure about an idea or have misconceptions		

	• **Reflect:**
	  ○ share the mathematical thinking of self and others, including evaluating strategies and solutions, extending, posing new problems and questions
	• **Connect mathematical concepts:**
	  ○ to develop a sense of how mathematics helps us understand ourselves and the world around us (e.g., daily activities, local and traditional practices, popular media and news events, social justice, cross-curricular integration)
	• **mistakes:**
	  ○ range from calculation errors to misconceptions
	• **opportunities to advance learning:**
	  ○ by:
	  – analyzing errors to discover misunderstandings
	  – making adjustments in further attempts
	  – identifying not only mistakes but also parts of a solution that are correct
	• **Incorporate:**
	  ○ by:
	  – collaborating with Elders and knowledge keepers among local		

	First Peoples – exploring the First Peoples Principles of Learning (http://www.fnesc.ca/wp/wp-content/uploads/2015/09/PUB-LFP-POSTER-Principles-of-Learning-First-Peoples-poster-11x17.pdf; e.g., Learning is holistic, reflexive, reflective, experiential, and relational [focused on connectedness, on reciprocal relationships, and a sense of place]; Learning involves patience and time) – making explicit connections with learning mathematics – exploring cultural practices and knowledge of local First Peoples and identifying mathematical connections • **knowledge:** ○ local knowledge and cultural practices that are appropriate to share and that are non-appropriated • **practices:** ○ Bishop's cultural practices: counting, measuring, locating, designing, playing, explaining (http://www.csus.edu/indiv/o/oreyd/ACP.htm_files/abishop.htm)		

Comment [mw1]: Possible to embed link in "First Peoples Principles of Learning" rather than showing URL?

	○ Aboriginal Education Resources (www.aboriginaleducation.ca) ○ *Teaching Mathematics in a First Nations Context*, FNESC (http://www.fnesc.ca/resources/math-first-peoples/)		